

Package ‘HadamardR’

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Type Package

Title Hadamard Matrix Generation

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Description

Generates Hadamard matrices using different construction methods. For those who want to generate Hadamard matrix, a generic function, `Hadamard_matrix()` is provided. For those who want to generate Hadamard matrix using a particular method, separate functions are available. See Horadam (2007, ISBN:9780691119212) Hadamard Matrices and their applications, Princeton University Press for more information on Hadamard Matrices.

License GPL-2

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| | |
|-------------------|--------------------------|
| HadamardR-package | <i>Hadamard Matrices</i> |
|-------------------|--------------------------|

Description

A square matrix H of order n with entries $+1$ or -1 is called Hadamard Matrix, if $HH' = nI$, where I is Identity matrix. Necessary condition for Hadamard matrix of order n exists when $n = 1, 2$ or $0 \pmod{4}$.

Details

Using HadamardR, Hadamard Matrices of various orders can be generated. Out of 1250 possible Hadamard Matrices of order < 5000 (ignoring trivial orders 1 and 2), the construction methods are not known for 45 orders; 1158 orders are possible using Hadamard_Matrix() function. 47 Hadamard matrices not available in this package are as follows: 1336, 1432, 1940, 2212, 2264, 2292, 2316, 2488, 2740, 2776, 2864, 2872, 3140, 3352, 3476, 3544, 3620, 3684, 3704, 3708, 3820, 3832, 3880, 3892, 3896, 3928, 3972, 3980, 4044, 4120, 4152, 4184, 4268, 4296, 4304, 4344, 4396, 4404, 4432, 4528, 4572, 580, 4632, 4740, 4792, 4812, 4976

| | |
|-------------|--------------------|
| antidiagnol | <i>antidiagnol</i> |
|-------------|--------------------|

Description

antidiagnol performs the creation of Back diagonol matrix.

Usage

antidiagnol(n)

Arguments

n integer

Details

An anti-diagonal matrix is a square matrix where all the entries are zero except those on the diagonal going from the lower left corner to the upper right corner entries are equal to 1.

In the first row, the last column will be 1 and all other entries are 0.

In second row, last but one column is 1 and others are 0 and so on.

Value

Antidiagonal matrix of order n.

Examples

```
antidiagnol(4)
#0  0  0  1
#0  0  1  0
#0  1  0  0
#1  0  0  0
```

baseseq

baseseq

Description

Extracts the selection of Basesequences from internal dataset. Not exported.

Usage

```
baseseq(order)
```

Arguments

order integer

Details

Create Basesequence of given order from the internal dataset Basesequence Base sequences are available in the internal table for order= 1:35

Value

Required Basesequences of order of x

Source

The Base sequences were obtained from [Christos Koukouvinos](#)

References

Goethals, J. M. and Seidel, J. J. (1967). Orthogonal matrices with zero diagonal. *Canad. J. Math.*, 19, 259-264.

 base_to_T

Base_to_T

Description

internal function and it is not exported. It converts base sequences to T-Sequences.

Usage

```
base_to_T(dat, x)
```

Arguments

| | |
|-----|---|
| dat | is the frame containing base sequences to be exported |
| x | integer (order of the base sequence) |

Details

dat - Internal dataset containing 4 sequences in long form with length $n+p, n+p, n, n$. Using the 4 base sequences, the function creates 4 sequences of length $2n+p, 2n+p, 2n+p, 2n+p$. T-Sequences are usually used in creating matrices of Goethel Seidal array.

Value

4 T-sequences of length of $2x+p$.

Source

The Base sequences were obtained from [Christos Koukouvinos](#)

References

Goethals, J. M. and Seidel, J. J. (1967). Orthogonal matrices with zero diagonal. *Canad. J. Math.*, 19, 259-264.

`cdn_baumert`*cdn_baumert*

Description

Checks Hadamard Matrix can be constructed using Baumert-Hall arrays of order 12.

Usage

```
cdn_baumert(order)
```

Arguments

`order` integer, order of Hadamard matrix to be checked.

Details

Baumert-Hall array is a generalization of Williamson Array. In case, Williamson matrices are available for order/12, the method return 6 otherwise it returns NULL.

The available Williamson sequences in the internal data sets is seq(1,63, 2) except 15, 35, 47, 53, 59 in the internal table.

Value

6 or NULL

References

Hedayat, A. and Wallis, W. D.(1978). Hadamard Matrices and Their Applications. Ann. Stat. 6: 1184-1238.

See Also

[had_baumert](#) for Baumert-Hall's construction method.

Examples

```
cdn_baumert(36)
#6
cdn_baumert(72)
#NULL
```

`cdn_cooper`*cdn_cooper*

Description

Checks Hadamard Matrix can be constructed using Williamson arrays and T- sequences.

Usage

```
cdn_cooper(order)
```

Arguments

order integer

Details

Cooper-Wallis is a construction of Hadamard matrices which combines Williamson matrices and T-sequences.

The available Williamson sequences in the internal data sets is seq(1,63, 2) except 15, 35, 47, 53, 59 in the internal table.

The available T- sequences in the internal data sets is seq(1,73,2) and 83, 101, 107.

Value

11 or NULL

References

Cooper, J., and Wallis, W., D. (1972). A construction for Hadamard arrays. Bull. Austral. Math. 7, 269-278.

See Also

[had_cooper](#) for Cooper-Wallis construction method. [get_cooper](#) for finding order of Williamson and T-Sequences.

Examples

```
cdn_cooper(20)
#11
cdn_cooper(16)
#NULL
```

`cdn_ehlich``cdn_ehlich`

Description

Checks Hadamard Matrix can be constructed using Ehlich's method.

Usage

```
cdn_ehlich(order)
```

Arguments

order integer

Details

Ehlich (1965)'s construction method requires order of the Hadamard matrix must be a of the form $(n-1)^2$. Conditions are (i) $Order=(n-1)^2$; (ii) n is a prime or prime power and $3 \pmod{4}$. (iii) $(n-2)$ must be a prime or prime power. In case, if all the three conditions are satisfied, function will return 4 or NULL.

Value

4 or NULL

References

Ehlich, H. (1965). Neue Hadamard-matrizen. Arch. Math., 16, 34-36.

See Also

[had_ehlich](#) for Ehlich's construction method.

Examples

```
cdn_ehlich(36)
#Condition 1:(n-1)^2 = 36 = 6^2
#Condition 2: n=7 (prime)and n=3(mod 4)
#Condition 3: n-2=5 (prime)
#Return
#4
cdn_ehlich(64)
#Condition 1:(n-1)^2 = 64 = 8^2
#Condition 2: n=9 (prime power) but n=1(mod 4).
#Condition 2 fails
#Return
#NULL
```

cdn_goethals_base *cdn_goethals_base*

Description

Checks Hadamard Matrix can be constructed using available base sequences.

Usage

```
cdn_goethals_base(order)
```

Arguments

order integer

Details

This function checks whether the Hadamard matrix of given order can be constructed using base sequences. If base sequences of length $n+1, n+1, n, n$ are available, T-sequences of length $2n+1, 2n+1, 2n+1, 2n+1$ can be constructed. From T-sequence of length $2n+1$, Hadamard matrix of order $4(2n+1)$ can be constructed. Returns the value 7, if it is possible otherwise NULL is returned.

Base sequences are available in the internal dataset is 1:35

Value

7 or NULL

Source

The Base sequences were obtained from [Christos Koukouvinos](#)

References

Goethals, J. M. and Seidel, J. J. (1967). Orthogonal matrices with zero diagonal. *Canad. J. Math.*, 19, 259-264.

See Also

[had_goethals_base](#) for Goethals-Seidel construction method.
[baseseq](#)

Examples

```
cdn_goethals_base(20)
#7
cdn_goethals_base(24)
#NULL
```

| | |
|----------------|-----------------------|
| cdn_goethals_T | <i>cdn_goethals_T</i> |
|----------------|-----------------------|

Description

Checks Hadamard Matrix can be constructed using available T-sequences.

Usage

```
cdn_goethals_T(order)
```

Arguments

| | |
|-------|---------|
| order | integer |
|-------|---------|

Details

This function checks whether the Hadamard matrix of given order can be constructed using T sequences. If T sequences of length n,n,n,n are available, Hadamard matrix of order $4n$ can be constructed. Returns the value 13, if it is possible otherwise NULL is returned.

T-sequences are available for length of seq(1,73,2) and for 83, 101 and 107 in the internal table.

Value

13 or NULL

See Also

[had_goethals_T](#) for Goethals-Seidel construction method using T-sequences.

Examples

```
cdn_goethals_T(28)
#T-sequence of length 7 exists.
#13
cdn_goethals_T(24)
#T-sequence of length 6 does not exist.
#NULL
```

| | |
|--------------------|---------------------------|
| cdn_goethals_Turyn | <i>cdn_goethals_Turyn</i> |
|--------------------|---------------------------|

Description

Checks Hadamard Matrix can be constructed using available Turyn Type sequences.

Usage

```
cdn_goethals_Turyn(order)
```

Arguments

| | |
|-------|---------|
| order | integer |
|-------|---------|

Details

This function checks whether the Hadamard matrix of given order can be constructed using Turyn sequences. If Turyn sequences of $(order+4)/12$ is available then Hadamard matrix of order exists. Returns the value 8, if it is possible otherwise NULL is returned.

Turyn type-sequences are available for 28,30,34,36 in the internal table.

Value

8 or NULL

See Also

[had_goethals_Turyn](#) for Goethals-Seidel construction method using Turyn sequences.

Examples

```
cdn_goethals_Turyn(356)
#8
cdn_goethals_Turyn(40)
#NULL
```

cdn_kronecker_matrix *cdn_kronecker_matrix*

Description

Checks Hadamard Matrix can be constructed by multiplying 2 existing Hadamard matrix.

Usage

```
cdn_kronecker_matrix(r)
```

Arguments

r integer

Details

This function checks whether the Hadamard matrix can be constructed as multiple of 2 Hadamard matrix. Returns the value 12, if it is possible otherwise NULL is returned.

Value

12 or NULL

Examples

```
cdn_kronecker_matrix(8)
#12
cdn_kronecker_matrix(12)
#NULL
```

cdn_miyamoto *cdn_miyamoto*

Description

Checks Hadamard Matrix can be constructed using Ehlich's method.

Usage

```
cdn_miyamoto(order)
```

Arguments

order integer

Details

In Miyamoto construction, if $q = n/4$ and q is a prime or prime power and $q \equiv 1 \pmod{4}$, then there exists an Hadamard Matrix of order n .

Value

9 or NULL

References

Miyamoto, M. (1991). A Construction of Hadamard matrices. J. Math. Phys., 12, 311-320.

See Also

[had_miyamoto](#) for Miyamoto's construction method.

Examples

```
cdn_miyamoto(20)
#q=5, is a prime number and q=1(mod 4).
#9
cdn_miyamoto(16)
#NULL
```

cdn_PaleyI

cdn_PaleyI Checks Hadamard Matrix can be constructed using Paley I method.

Description

cdn_PaleyI Checks Hadamard Matrix can be constructed using Paley I method.

Usage

```
cdn_PaleyI(order)
```

Arguments

order integer

Details

In Paley I method, if $q = \text{order} - 1$ and q is prime number and $q \equiv 3 \pmod{4}$ then the function returns 2 otherwise NULL.

Value

2 or NULL

References

Paley, R.E.A.C. (1933). On Orthogonal matrices. J. Combin. Theory, A 57(1), 86-108.

See Also

[PaleyI](#) for Paley I construction method.

Examples

```
cdn_PaleyI(8)
#2
cdn_PaleyI(16)
#NULL
```

| | |
|-------------|---|
| cdn_PaleyII | <i>cdn_PaleyII Checks Hadamard Matrix can be constructed using Paley II method.</i> |
|-------------|---|

Description

cdn_PaleyII Checks Hadamard Matrix can be constructed using Paley II method.

Usage

```
cdn_PaleyII(order)
```

Arguments

| | |
|-------|---------|
| order | integer |
|-------|---------|

Details

In Paley II method, If $q = \text{order}/2 - 1$ or $q = \text{order}/4 - 1$ and q is prime number and $q \equiv 1 \pmod{4}$ then this function returns 3 otherwise NULL.

Value

3 or NULL

References

Paley, R.E.A.C. (1933). On Orthogonal matrices. J. Combin. Theory, A 57(1), 86-108.

See Also

[PaleyII](#) for Paley II construction method.

Examples

```
cdn_PaleyII(24)
#3
cdn_PaleyII(16)
#NULL
```

cdn_PaleyIIprimepower *cdn_PaleyIIprimepower checks Hadamard Matrix can be constructed using Paley II method.*

Description

cdn_PaleyIIprimepower checks Hadamard Matrix can be constructed using Paley II method.

Usage

```
cdn_PaleyIIprimepower(order)
```

Arguments

order integer

Details

In Paley II method, $q = \text{order}/2 - 1$ and q is prime power and $q \equiv 1 \pmod{4}$ then it returns 15 otherwise NULL.

Value

15 or NULL

References

Paley, R.E.A.C. (1933). On Orthogonal matrices. J. Combin. Theory, A 57(1), 86-108.

See Also

[PaleyIIPrimePower](#) for Paley construction method.

Examples

```
cdn_PaleyIIprimepower(340)
#15
cdn_PaleyIIprimepower(64)
#NULL
```

cdn_PaleyIprimepower *cdn_PaleyIprimepower checks Hadamard Matrix can be constructed using Paley I method.*

Description

cdn_PaleyIprimepower checks Hadamard Matrix can be constructed using Paley I method.

Usage

```
cdn_PaleyIprimepower(order)
```

Arguments

order integer

Details

In Paley I method, If $q = \text{order} - 1$ and q is prime power and $q \equiv 3 \pmod{4}$ then it returns 14 otherwise NULL.

Value

14 or NULL

References

Paley, R.E.A.C. (1933). On Orthogonal matrices. J. Combin. Theory, A 57(1), 86-108.

See Also

[PaleyIPrimePower](#) for Paley I construction method.

Examples

```
cdn_PaleyI(28)
#14
cdn_PaleyI(16)
#NULL
```

| | |
|---------|----------------|
| cdn_sds | <i>cdn_sds</i> |
|---------|----------------|

Description

Checks Hadamard Matrix can be constructed using available Supplementary Difference Sets.

Usage

```
cdn_sds(order)
```

Arguments

order integer

Details

This function checks whether the Hadamard matrix of given order can be constructed using Supplementary Difference sets. If SDS is available it Returns the value 10 otherwise NULL.

SDS are available for 103,127,151,163,181,191,239,251,463,631 in the internal table.

Value

10 or NULL

Source

SDS sets are available from Djokovic (1992a,b,c,d and 1994a,1994b).

References

- Djokovic, D. Z. (1992a). Skew Hadamard matrices of order 4x37 and 4x39. J. Combin. Theory, A 61, 319-321.
- Djokovic, D. Z. (1992b). Construction of some new Hadamard matrices. Bull. Austral. math. Soc., 45, 327-332.
- Djokovic, D. Z. (1992c). Ten new Hadamard matrices of skew type. Publ.Electrotechnickog Fak., Ser. Matematika, Univ. of Belgrade, 3, 47-59.
- Djokovic, D. Z. (1992d). Ten Hadamard matrices of order 1852 of Goethals-Seidel type. Europ. J. Combinatorics, 13, 245-248.
- Djokovic, D. Z. (1994a). Two Hadamard matrices of order 956 of Goethals-Seidel type. Combinatorica, 14(3), 375-377.
- Djokovic, D. Z. (1994b). Five new Hadamard matrices of order skew type. Austral. J. Combinatorics, 10, 259-264.

See Also

[had_SDS](#) for SDS construction method.

Examples

```
cdn_sds(412)
#10
cdn_sds(428)
#NULL
```

| | |
|----------------|-----------------------|
| cdn_williamson | <i>cdn_williamson</i> |
|----------------|-----------------------|

Description

Checks Hadamard Matrix can be constructed using available Williamson sequences.

Usage

```
cdn_williamson(order)
```

Arguments

order integer

Details

This function checks whether the Hadamard matrix of given order can be constructed using williamson sequences. If Williamson sequences of length n,n,n,n are available, Hadamard matrix of order $4n$ can be constructed. Returns the value 5, if it is possible otherwise NULL is returned.

Williamson sequences are available for length of seq(1,63, 2) except 15, 35, 47, 53, 59 in the internal table.

Value

5 or NULL

Source

The Williamson sequences were obtained from [Christos Koukouvinos](#) and London (2013).

References

- Williamson, J. (1944). Hadamard determinant theorem and the sum of four squares. Duke. Math. J., 11, 65-81.
- Williamson, J. (1947). Note on Hadamard's determinant theorem. Bull. Amer. Math. Soc., 53, 608-613.
- London, S. 2013. Constructing New Turyn Type Sequences, T-Sequences and Hadamard Matrices. PhD Thesis, University of Illinois at Chicago, Chicago.

See Also

[had_williamson](#) for Williamson construction method using Williamson sequences.

Examples

```
cdn_williamson(20)
#5
cdn_goethals_T(24)
#NULL
```

| | |
|----------------|-----------------------|
| check_hadamard | <i>check_hadamard</i> |
|----------------|-----------------------|

Description

check_hadamard tests whether the input matrix is an Hadamard matrix or not.

Usage

```
check_hadamard(x)
```

Arguments

x matrix

Details

This function can be used to check whether a given matrix is an Hadamard Matrix or not. To ensure that generated matrix is indeed an Hadamard matrix, this function can be used. In case, if the given matrix is an Hadamard matrix, a text message, Given matrix is an Hadamard Matrix of order is printed on the console.

This function checks (i)Input is a matrix; (ii)a square matrix; (iii)Order of the matrix is an Hadamard number; (iv) All elements are either +1 or -1; (v) $HH' = nI$, where n is the order of the input matrix H and H' is transpose of H.

Value

returns a text message

References

Hedayat, A. and Wallis, W.D. (1978). Hadamard Matrices and Their Application. Ann. Stat., 6, 1184-1238.

Examples

```

#Example 1:
h<-matrix(c(1,1,1,-1),nrow=2,ncol=2)
check_hadamard(h)
# Given matrix is an Hadamard Matrix of order 2
#Example 2:
h<-matrix(c(1,-1,1,-1),nrow=2,ncol=2)
check_hadamard(h)
#Not an Hadamard matrix
#Example 3:
h<-Hadamard_Matrix(36)
check_hadamard(h)
#"Given matrix is an Hadamard Matrix of order 36"

```

circulant_mat

circulant_mat

Description

A matrix is said to be circulant if $(i+1, j+1)$ th entry is equal to the (i, j) th entry. Thus, for such matrices, the initial row determines the complex matrix. Whenever $i+1, j+1$ exceeds the order, modulus operation is carried out.

Usage

```
circulant_mat(x = NA)
```

Arguments

x a vector to be used as initial row.

Details

circulant_mat performs construction of circulant matrices.

Value

circulant matrix of order length of input vector.

References

Hedayat, A. and Wallis, W.D. (1978). Hadamard Matrices and Their Application. Ann. Stat., 6, 1184-1238.

Examples

```

circulant_mat(c(1,1,-1,0))
#      [,1] [,2] [,3] [,4]
#[1,]  1   1  -1   0
#[2,]  0   1   1  -1
#[3,] -1   0   1   1
#[4,]  1  -1   0   1
circulant_mat(c(5,9,-7,-2))
#      [,1] [,2] [,3] [,4]
#[1,]  5   9  -7  -2
#[2,] -2   5   9  -7
#[3,] -7  -2   5   9
#[4,]  9  -7  -2   5

```

```
get_cooper
```

```
get_cooper
```

Description

This function provides the Williamson Matrix order and T-Sequence length required to construct Hadamard matrix.

Usage

```
get_cooper(x)
```

Arguments

x integer Hadamard Matrix Order to Check

Details

If m is the order of T-Sequence and n is the order of Williamson sequence and both exists. Cooper and Wallis (1972) showed a construction method for Hadamard matrix of order 4mn exists. This function returns m and n if they exists otherwise NULL value is returned.

Value

m Tsequence order
n Williamson order

References

Cooper, J., and Wallis, J. 1972. A construction for Hadamard arrays. Bull. Austral. Math. Soc., 07: 269-277.

Examples

```

get_cooper(340)
#$m
#[1] 5
#$n
#[1] 17
get_cooper(256)
#NULL

```

Get_method

Get_method

Description

Get_method helps finding the given order of the matrix is constructed by which method.

Usage

```
Get_method(order)
```

Arguments

```
order          integer
```

Value

Method name of the given order.

Examples

```

Get_method(92) # Williamson method
Get_method(24)
# Paley I

```

GFADD

GFADD

Description

Addition table of GF(P^r)

Usage

```
GFADD(GFElem, p, r)
```

Arguments

| | |
|--------|--|
| GFElem | integer (Can be obtained by calling GFELEM function) |
| p | integer (a prime number) |
| r | integer (a positive integer) |

Details

This function returns addition table of Galois field of order p^r . To use this function, Minimum function, elements of GF are required. Minimum functions are available in internal dataset. Elements can be generated using GFELEM function.

Value

A matrix of size $p^r \times p^r$

Examples

```
p<-3
r<-2
cardin<-p^2
mf<-subset(HadamardR:::minimumfunction,HadamardR:::minimumfunction$s==cardin)
MF<-mf$coeff
GFElem<-GFELEM(p,r,MF)
GFADD(GFElem,p,r)
#Addition Table of GF(9)
```

GFCheck

GFCheck

Description

This is an internal function to return the position of argument add in elements of GF(cardin)

Usage

```
GFCheck(GFElem, r, cardin, add)
```

Arguments

| | |
|--------|---------------|
| GFElem | integer array |
| r | integer |
| cardin | integer |
| add | integer array |

Details

This function is not exported. Used for checking the result of addition or multiplication of GFElements.

Value

`i` integer The position of the element checked in GFElem

 GFELEM

GFELEM

Description

Elements of Galois Field, $GF(P^r)$

Usage

GFELEM(`p`, `r`, MF)

Arguments

| | |
|----------------|---|
| <code>p</code> | integer (a prime number) |
| <code>r</code> | integer (a positive integer) |
| MF | Integer Array containing Minimum function |

Details

This function returns Elements of Galois field of order p^r . To use this function, Minimum function is required. Minimum functions are available in internal dataset. To use the Minimum function from the internal, use HadamardR:::

Value

A vector of size p^r

Examples

```
library(HadamardR)
p<-3
r<-2
cardin=9
mf<-subset(HadamardR:::minimumfunction,HadamardR:::minimumfunction$s==cardin)
MF<-mf$coeff
GFElem<-GFELEM(p,r,MF)
GFElem
```

| | |
|-----|--|
| GFM | <i>GFM GFM Generate Multiplication table of GF(p^r), where p is a prime power.</i> |
|-----|--|

Description

GFM GFM Generate Multiplication table of GF(p^r), where p is a prime power.

Usage

GFM(cardin)

Arguments

cardin integer

Details

This function returns Multiplication table of Galois field of order p^r. To use this function, Minimum function, elements of GF are required. Minimum functions are available in internal dataset. Elements can be generated using GFELEM function.

Value

Multiplication table of GF(p^r)

Examples

```
GFM(9)
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
## [1,]  1   1   1   1   1   1   1   1   1
## [2,]  1   3   4   5   6   7   8   9   2
## [3,]  1   4   5   6   7   8   9   2   3
## [4,]  1   5   6   7   8   9   2   3   4
## [5,]  1   6   7   8   9   2   3   4   5
## [6,]  1   7   8   9   2   3   4   5   6
## [7,]  1   8   9   2   3   4   5   6   7
## [8,]  1   9   2   3   4   5   6   7   8
## [9,]  1   2   3   4   5   6   7   8   9
```

| | |
|--------|--|
| GFMult | <i>GFMult GFMult Generate Multiplication table of GF(p^r), where p is a prime power.</i> |
|--------|--|

Description

GFMult GFMult Generate Multiplication table of GF(p^r), where p is a prime power.

Usage

GFMult(cardin)

Arguments

cardin integer

Details

This function returns Multiplication table of Galois field of order p^r. To use this function, Minimum function, elements of GF are required. Minimum functions are available in internal dataset. Elements can be generated using GFELEM function.

Value

Multiplication table of GF(p^r)

Examples

```
GFMult(9)
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
## [1,]  1   1   1   1   1   1   1   1   1
## [2,]  1   3   4   5   6   7   8   9   2
## [3,]  1   4   5   6   7   8   9   2   3
## [4,]  1   5   6   7   8   9   2   3   4
## [5,]  1   6   7   8   9   2   3   4   5
## [6,]  1   7   8   9   2   3   4   5   6
## [7,]  1   8   9   2   3   4   5   6   7
## [8,]  1   9   2   3   4   5   6   7   8
## [9,]  1   2   3   4   5   6   7   8   9
```

| | |
|------------|-------------------|
| GFPrimeAdd | <i>GFPrimeAdd</i> |
|------------|-------------------|

Description

GFPrimeAdd creates the addition Table for GF(p), where p is a prime number

Usage

GFPrimeAdd(p)

Arguments

p integer

Details

If the elements of GF(p) are 0,1,...,p-1 then the (i,j)th element of matrix returned is addition of (i-1)th and (j-1)th elements. The additions are subjected to modulo p.

Value

Addition Table of GF(p) in the form of matrix of order p x p.

Examples

```
GFPrimeAdd(5)
#[,1] [,2] [,3] [,4] [,5]
#[1,]  0  1  2  3  4
#[2,]  1  2  3  4  0
#[3,]  2  3  4  0  1
#[4,]  3  4  0  1  2
#[5,]  4  0  1  2  3
```

| | |
|-------------|--|
| GFPrimeMult | <i>GFPrimeMult GFPrimeMult creates Multiplication Table for GF(p), where p is a prime number</i> |
|-------------|--|

Description

GFPrimeMult GFPrimeMult creates Multiplication Table for GF(p), where p is a prime number

Usage

```
GFPrimeMult(p)
```

Arguments

p integer

Details

If the elements of GF(p) are 0,1,...,p-1 then the (i,j)th element of matrix returned is multiplication of (i-1)th and (j-1)th elements. The multiplications are subjected to modulo p.

Value

Multiplication Table of GF(p) in the form of matrix of order p x p.

Examples

```
GFPrimeMult(5)
#[,1] [,2] [,3] [,4] [,5]
#[1,]  0  0  0  0  0
#[2,]  0  1  2  3  4
#[3,]  0  2  4  1  3
#[4,]  0  3  1  4  2
#[5,]  0  4  3  2  1
```

goethals_seidel_array *goethals_seidel_array*

Description

goethals_seidel_array performs the construction of Hadamard matrix by Goethals-Seidel method

Usage

```
goethals_seidel_array(A = NA, B = NA, C = NA, D = NA)
```

Arguments

A matrix
 B matrix
 C matrix
 D matrix

Details

For this function requires the four matrices, all the four matrix are Circulant matrices same order. R is an antidiagonal matrix of the same order With which it should satisfy the $AA' + BB' + CC' + DD' = 4nI$, where I is the identity matrix of order n . This function returns matrix of order $4n$ where n is the order of the given matrices.

Value

goethals_seidel matrix of order $4n$

References

Goethals, J. M. and Seidel, J. J. (1967). Orthogonal matrices with zero diagonal. *Canad. J. Math.*, 19, 259-264.

| | |
|-----------------|------------------------|
| Hadamard_Matrix | <i>Hadamard_Matrix</i> |
|-----------------|------------------------|

Description

Hadamard_Matrix is generic function for construction of Hadamard matrix.

Usage

Hadamard_Matrix(order)

Arguments

order integer

Details

function Hadamard_matrix was created which does not require known of construction methods. Hadamard_matrix() takes an integer as input and returns Hadamard matrix if it is available. In case, it is not possible to construct, NULL value is returned.

Value

Hadamard Matrix of given Order

Examples

```

Hadamard_Matrix(1)
#1
Hadamard_Matrix(2)
#      [,1] [,2]
# [1,]    1    1
# [2,]    1   -1
Hadamard_Matrix(8)
#      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
# [1,]    1    1    1    1    1    1    1    1
# [2,]    1   -1    1   -1    1   -1    1   -1
# [3,]    1    1   -1   -1    1    1   -1   -1
# [4,]    1   -1   -1    1    1   -1   -1    1
# [5,]    1    1    1    1   -1   -1   -1   -1
# [6,]    1   -1    1   -1   -1    1   -1    1
# [7,]    1    1   -1   -1   -1   -1    1    1
# [8,]    1   -1   -1    1   -1    1    1   -1
Hadamard_Matrix(10)
#"Order is not a Hadamard number"
Hadamard_Matrix(668)
#"Not possible to construct or order is not a multiple of 4"

```

Hadamard_matrix_method

Hadamard_Matrix_method

Description

Hadamard_Matrix_method it is also generic function but it provides some additional options.

Usage

```
Hadamard_matrix_method(order, type = -1, method = "", file = "", filetype = "")
```

Arguments

| | |
|----------|--|
| order | integer |
| type | -1 or 0 |
| method | method=c("Kronecker", "PaleyI", "PaleyII", "Ehlich", "Williamson", "Baumert", "Goethals-Seidel_Base", "Goethals-Seidel_Turyn", "Miyamoto", "Cooper-Wallis", "Kronecker_Product_Method", "P |
| file | Name of the file |
| filetype | xlsx or csv |

Details

If the method is not specified or incorrectly specified, Hadamard matrix will be constructed using Had_method function. If the method is specified, Hadamard matrix will be constructed using that method.

By default, the elements will be +1 or -1. In case, -1 should be replaced by 0, use type=0.

TO save the generated matrix into a text file (csv) or MS-Excel, filename may be specified (with extension). In case Excel file required, use filetype = xlsx, otherwise csv file will be generated.

If just give the input as number it returns Hadamard matrix in console.

Value

Hadamard Matrix of given Order

Examples

```
Hadamard_matrix_method(4)
#      [,1] [,2] [,3] [,4]
#[1,]  1   1   1   1
#[2,]  1  -1   1  -1
#[3,]  1   1  -1  -1
#[4,]  1  -1  -1   1
Hadamard_matrix_method(8,method = "PaleyI")
#      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
# [1,]  1   1   1   1   1   1   1   1
# [2,] -1   1  -1  -1   1  -1   1   1
# [3,] -1   1   1  -1  -1   1  -1   1
# [4,] -1   1   1   1  -1  -1   1  -1
# [5,] -1  -1   1   1   1  -1  -1   1
# [6,] -1   1  -1   1   1   1  -1  -1
# [7,] -1  -1   1  -1   1   1   1  -1
# [8,] -1  -1  -1   1  -1   1   1   1
```

```
Hadamard_matrix_method(12,method = "Williamson",
  file = file.path(tempdir(), "Hadamard12.csv"))
#output saved in file
```

```
Hadamard_matrix_method(36,method = "Baumert",
  file = file.path(tempdir(), "Hadamard36.xlsx"))
#output saved in file
```

```
Hadamard_matrix_method(20,method = "Miyamoto",
  file = file.path(tempdir(), "Hadamard20.csv"),filetype = "csv")
#output saved in file
```

```
Hadamard_matrix_method(8,method =
  "Kronecker",file = file.path(tempdir(), "Hadamard8.xlsx"), filetype = "xlsx")
#output saved in file
```

| | |
|-------------|--------------------|
| had_baumert | <i>had_baumert</i> |
|-------------|--------------------|

Description

had_baumert performs the construction of Hadamard matrix by Baumert-Hall method.

Usage

```
had_baumert(n)
```

Arguments

n integer (order of the matrix)

Details

Baumert-Hall arrays extension of the williamson array. For construction of matrix it requires the Williamson sequences. For different order of the matrix it requires different williamson sequences. If williamson sequences are not available it Returns NULL.

Williamson sequences are available for length of seq(1,63, 2) except 15, 35, 47, 53, 59 in the internal table.

Value

Hadamard matrix of order n

Source

The Williamson sequences are available in London (2013) and [Christos Koukouvinos](#)

References

Baumert, L. D., and Hall, M. Jr. (1965). A new construction method for Hadamard matrices. Bull. Amer. Math. Soc., 71, 169-170

Hedayat, A. and Wallis, W.D. (1978). Hadamard Matrices and Their Application. Ann. Stat., 6, 1184-1238.

London, S. 2013. Constructing New Turyn Type Sequences, T-Sequences and Hadamard Matrices. PhD Thesis, University of Illinois at Chicago, Chicago.

Examples

```
had_baumert(372)

#Big matrix.
had_baumert(24)
#NULL
```


Details

Ehlich (1965)'s construction method requires order of the Hadamard matrix must be a of the form $(n-1)^2$. Conditions are (i) $\text{Order}=(n-1)^2$; (ii) n is a prime or prime power and $3 \pmod{4}$; (iii) $(n-2)$ must be a prime or prime power. In case, if all the three conditions are satisfied, then function will return Hadamard matrix of order x otherwise NULL.

Value

Hadamard matrix of order x

References

Ehlich, H. (1965). Neue Hadamard-matrizen. Arch. Math., 16, 34-36.

Examples

```
had_ehlich(36)
had_ehlich(20)
#NULL
```

had_goethals_base *had_goethals_base*

Description

had_goethals_base performs the construction of Hadamard Matrix from Goethals-Seidel method. by using the Base sequences.

Usage

```
had_goethals_base(x)
```

Arguments

x integer (order of the matrix)

Details

This function construct the Hadamard matrix of given order using base sequences. If base sequences of length $n+1, n+1, n, n$ are available, base sequences are converted into T-sequences of length $2n+1, 2n+1, 2n+1, 2n+1$ can be constructed. From T-sequence of length $2n+1$, Hadamard matrix of order $4(2n+1)$ can be constructed. For a given order the base sequences is not available it returns NULL.

The Base sequences are stored in internal dataset. The available Base sequences of length is 1,2,3,4,.....,35

Value

Hadamard matrix of order x

Source

The Base sequences were obtained from [Christos Koukouvinos](#)

References

Goethals, J. M. and Seidel, J. J. (1967). Orthogonal matrices with zero diagonal. *Canad. J. Math.*, 19, 259-264.

Examples

```
had_goethals_base(12)
#      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12]
# [1,]  1   1   1   1   1   -1   1   -1   -1   -1   1   -1
# [2,]  1   1   1   1   -1   1   -1   -1   1   1   1   -1
# [3,]  1   1   1   -1   1   1   -1   1   -1   -1   -1   1
# [4,] -1  -1   1   1   1   1   1   -1   -1   1   -1   1
# [5,] -1   1  -1   1   1   1   -1   -1   1   -1   1   1
# [6,]  1  -1  -1   1   1   1   -1   1   -1   1   1   -1
# [7,] -1   1   1  -1   1   1   1   1   1   1   1   -1
# [8,]  1   1  -1   1   1  -1   1   1   1   1   -1   1
# [9,]  1  -1   1   1  -1   1   1   1   1   -1   1   1
#[10,]  1  -1   1  -1   1  -1  -1  -1  -1   1   1   1
#[11,] -1   1   1   1  -1  -1  -1   1  -1   1   1   1
#[12,]  1   1  -1  -1  -1   1   1  -1  -1   1   1   1
had_goethals_base(16)
#NULL
```

```
had_goethals_T      had_goethals_T had_goethals_Turyn performs the Hadamard Matrix
                    from Goethals-Seidel method by using T sequences.
```

Description

had_goethals_T had_goethals_Turyn performs the Hadamard Matrix from Goethals-Seidel method by using T sequences.

Usage

```
had_goethals_T(n)
```

Arguments

n integer (order of the matrix)

Details

This function construct Hadamard matrix of given order using T sequences. If T sequences of length n, n, n, n are available, Hadamard matrix of order $4n$ can be constructed. Returns the Hadamard matrix of given order. If for given order the T sequences are not available returns NULL.

The T sequences are stored in internal dataset. The available T sequences of length is seq(1,73,2) and 83, 101 and 107

Value

Hadamard matrix of order x

Source

The T sequences are available at London (2013) and The Base sequences were obtained from [Christos Koukouvinos](#)

References

Goethals, J. M. and Seidel, J. J. (1967). Orthogonal matrices with zero diagonal. *Canad. J. Math.*, 19, 259-264.

London, S. 2013. Constructing New Turyn Type Sequences, T-Sequences and Hadamard Matrices. PhD Thesis, University of Illinois at Chicago, Chicago.

Examples

```
had_goethals_T(4)
#      [,1] [,2] [,3] [,4]
# [1,]  1  -1  -1  -1
# [2,]  1   1  -1   1
# [3,]  1   1   1  -1
# [4,]  1  -1   1   1
had_goethals_T(8)
#NULL
```

`had_goethals_Turyn` *had_goethals_Turyn*

Description

`had_goethals_Turyn` performs the Hadamard Matrix from Goethals-Seidel method by using Turyn sequences.

Usage

`had_goethals_Turyn(r)`

Arguments

`r` integer (order of the matrix)

Details

This function construct Hadamard matrix of given order using Turyn sequences. If Turyn sequences of length $2n-1$, $2n-1$, n , n is available then Turyn sequences are converted in T sequences of length $2n+p$, $2n+p$, $2n+p$, $2n+p$ and $p=n-1$, these T sequences are used for construction of Hadamard matrix. If the given order of the the Turyn sequences are not available it returns NULL.

Turyn type-sequences are available for 28,30,34,36 in the internal dataset.

Value

Hadamard matrix of order `r`

Source

The Base sequences were obtained from [Christos Koukouvinos](#)

References

Goethals, J. M. and Seidel, J. J. (1967). Orthogonal matrices with zero diagonol. Canad. J. Math., 19, 259-264.

Examples

```
#Big matrices
had_goethals_Turyn(356)
had_goethals_Turyn(404)
```

| | |
|----------------------------|----------------------|
| <code>had_kronecker</code> | <i>had_kronecker</i> |
|----------------------------|----------------------|

Description

`had_kronecker` performs the construction of an Hadamard matrix by kronecker product.

Usage

```
had_kronecker(n, exponent = NULL)
```

Arguments

`n` an integer (Expected to be Hadamard Number)
`exponent` an integer

Details

This function only applicable when n is the power of 2 and multiple of 4.

If n<-2, returns Hadamard matrix of order 2; if n is not Hadamard number, return NULL.

By default exponent=FALSE; when exponent is unknown it is computed.

If exponent is given use the same

Value

Hadamard Matrix of order n, if n is power of 2, otherwise NULL.

References

Hedayat, A. and Wallis, W.D. (1978). Hadamard Matrices and Their Application. Ann. Stat., 6, 1184-1238.

Sylvester, J.J. (1968). Problem 2511. Math. Questions and solutions, 10, 74.

Examples

```
had_kronecker(4)
#      [,1] [,2] [,3] [,4]
#[1,]  1   1   1   1
#[2,]  1  -1   1  -1
#[3,]  1   1  -1  -1
#[4,]  1  -1  -1   1
had_kronecker(8,3)
#      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
#[1,]  1   1   1   1   1   1   1   1
#[2,]  1  -1   1  -1   1  -1   1  -1
#[3,]  1   1  -1  -1   1   1  -1  -1
#[4,]  1  -1  -1   1   1  -1  -1   1
#[5,]  1   1   1   1  -1  -1  -1  -1
#[6,]  1  -1   1  -1  -1  -1   1  -1
#[7,]  1   1  -1  -1  -1  -1   1   1
#[8,]  1  -1  -1   1  -1   1   1  -1
had_kronecker(9)
# NULL
```

Had_method

Had_method

Description

Had_method performs the give order of the matrix is constructed by which method.

Usage

Had_method(order)

Arguments

order integer (order of the Hadamard matrix)

Details

- If the method number returns, if it
- 1 kronecker method (power of 2 only)
 - 2 PaleyI
 - 3 PaleyII
 - 4 Ehlich method
 - 5 Williamson method
 - 6 Baumert-Hall method
 - 7 Goethals-Seidel by using Base sequences
 - 8 Goethals-Seidel by using Turyn sequences
 - 9 Miyamoto method
 - 10 Suplimentary Difference Sets
 - 11 Cooper-Wallis method
 - 12 Kronecker product method
 - 13 Goethals-Seidel by using T sequences
 - 14 Paley I Prime Power
 - 15 Paley II Prime Power

Value

Method number

Examples

```
Had_method(92) # "5"  
Had_method(324) # "4"
```

had_miyamoto *had_miyamoto*

Description

had_miyamoto function perform the construction of the Hadamard matrix by using the Miyamoto method

Usage

had_miyamoto(n)

Arguments

n integer (order of the matrix)

Details

If the $q=n/4$, and q be a prime power and $q \equiv 1 \pmod{4}$. If there is a exists of Hadamard matrix of order $q-1$, then there exists an Hadamard matrix of order $4q$. If given order is not satisfied it returns NULL.

Value

Hadamard matrix of n

References

Miyamoto, M. (1991). A Construction of Hadamard matrices. J. Math. Phy., 12, 311-320.

Examples

```
had_miyamoto(20)
had_miyamoto(24) #NULL
```

had_SDS

had_SDS

Description

had_SDS performs the construction of Hadamard matrix from SDS.

Usage

```
had_SDS(x)
```

Arguments

x integer (order of the matrix)

Details

This function construct the Hadamard matrix of given order can be constructed using Supplementary Diffrence sets. For given order the SDS set is not available it returns NULL If SDS is available it Returns Hadamard matrix of given order.

SDS are available for 103,127,151,163,181,191,239,251,463,631 in the internal table.

Value

Hadamard matrix of order x

References

- Djokovic, D. Z. (1992a). Skew Hadamard matrices of order 4×37 and 4×39 . J. Combin. Theory, A 61, 319-321.
- Djokovic, D. Z. (1992b). Construction of some new Hadamard matrices. Bull. Austral. math. Soc., 45, 327-332.
- Djokovic, D. Z. (1992c). Ten new Hadamard matrices of skew type. Publ. Electrotechnickog Fak., Ser. Matematika, Univ. of Belgrade, 3, 47-59.
- Djokovic, D. Z. (1992d). Ten Hadamard matrices of order 1852 of Goethals-Seidel type. Europ. J. Combinatorics, 13, 245-248.
- Djokovic, D. Z. (1994a). Two Hadamard matrices of order 956 of Goethals-Seidel type. Combinatorica, 14(3), 375-377.
- Djokovic, D. Z. (1994b). Five new Hadamard matrices of order skew type. Austral. J. Combinatorics, 10, 259-264.

Examples

```
had_SDS(412)
```

```
had_SDS(508)
```

| | |
|----------------|-----------------------|
| had_williamson | <i>had_williamson</i> |
|----------------|-----------------------|

Description

had_williamson performs the construction Hadamard matrix from Williamson method by using the williamson sequences.

Usage

```
had_williamson(x)
```

Arguments

x integer (order of the matrix)

Details

This function construct Hadamard matrix of given order using williamson sequences. If Williamson sequences of length n, n, n, n are available, Hadamard matrix of order $4n$ can be constructed. If for given order of Matrix Williamson sequences are not available it returns NULL.

The Williamson sequences are stored in internal dataset, available for length of seq(1,63, 2) except 15, 35, 47, 53, 59 in the internal table.

Value

Hadamard matrix

Source

The williamson sequences are available in London(2013) and [Christos Koukouvinos](#)

References

Williamson, J. (1944). Hadamard determinant theorem and the sum of four squares. Duke. Math. J., 11, 65-81.

Williamson, J. (1947). Note on Hadamard's determnant theorem. Bull. Amer. Math. Soc., 53, 608-613.

London, S. 2013. Constructing New Turyn Type Sequences, T-Sequences and Hadamard Matrices. PhD Thesis, University of Illinois at Chicago, Chicago.

Examples

```
had_williamson(4)
#      [,1] [,2] [,3] [,4]
#[1,]   1   1   1   1
#[2,]  -1   1  -1   1
#[3,]  -1   1   1  -1
#[4,]  -1  -1   1   1
had_williamson(8)
# NULL
```

Initial_row_SDS *Initial_row_SDS Initial_row_SDS is an internal function.Not Ex-ported.*

Description

Initial_row_SDS Initial_row_SDS is an internal function.Not Exported.

Usage

```
Initial_row_SDS(i, j, v, n, r)
```

Arguments

| | |
|---|------------------------|
| i | is the numeric vectors |
| j | is the numeric vectors |
| v | is the numeric vectors |
| n | is the numeric vectors |
| r | is the numeric vectors |

Details

All inputs are numeric vectors of same length. This function used in the CONstruction of Hadamard matrix by Supplementary Differences Sets It converts the SDS sets into binary forms (+1 or -1).

Value

Intial rows of Matrix.

References

Djokovic, D. Z. (1992a). Skew Hadamard matrices of order 4×37 and 4×39 . J. Combin. Theory, A 61, 319-321.

Djokovic, D. Z. (1992b). Construction of some new Hadamard matrices. Bull. Austral. math. Soc., 45, 327-332.

Djokovic, D. Z. (1992c). Ten new Hadamard matrices of skew type. Publ.Electrotechnickog Fak., Ser. Matematika, Univ. of Belgrade, 3, 47-59.

Djokovic, D. Z. (1992d). Ten Hadamard matrices of order 1852 of Goethals-Seidel type. Europ. J. Combinatorics, 13, 245-248.

Djokovic, D. Z. (1994a). Two Hadamard matrices of order 956 of Goethals-Seidel type. Combinatorica, 14(3), 375-377.

Djokovic, D. Z. (1994b). Five new Hadamard matrices of order skew type. Austral. J. Combinatorics, 10, 259-264.

 is.prime

is.prime

Description

is.prime check the given number is prime or not

Usage

```
is.prime(num)
```

Arguments

num integer

Details

if the given number is divisible any number other than 1 and itself it return NULL. otherwise TRUE.

Value

TRUE or FALSE

Examples

```
is.prime(3)
#TRUE
is.prime(21)
#FALSE
```

is.primepower

is.primepower

Description

Checks whether given number is a prime power or not. Note that for a prime number, it would return NULL.

Usage

```
is.primepower(p)
```

Arguments

p integer

Details

Returns *a* and *b* where $p=a^b$, otherwise NULL. Uses `primeFactors()` function of `numbers` package.

Value

a and *b* where $p=a^b$ and *a* is a prime number. Otherwise NULL

Examples

```
is.primepower(2048)
#2 11
is.primepower(7)
#NULL
is.primepower(100)
#NULL
```

| | |
|--------------|---------------------|
| is_divisible | <i>is_divisible</i> |
|--------------|---------------------|

Description

is_divisible is internal function. Not exported.

Usage

```
is_divisible(num, divisor)
```

Arguments

| | |
|---------|---------|
| num | integer |
| divisor | integer |

Details

it returns num/divisor value.

Value

num/divisor

| | |
|------|-------------|
| Jmat | <i>Jmat</i> |
|------|-------------|

Description

Jmat performs the generation of unit matrix.

Usage

```
Jmat(n)
```

Arguments

| | |
|---|---------|
| n | integer |
|---|---------|

Details

An J matrix is a square matrix where all the entries are one.

Value

square matrix of order n

Examples

```
Jmat(4)
#      [,1] [,2] [,3] [,4]
#[1,]  1   1   1   1
#[2,]  1   1   1   1
#[3,]  1   1   1   1
#[4,]  1   1   1   1
```

```
kronecker_matrix      kronecker_matrix
```

Description

kronecker_matrix

Usage

```
kronecker_matrix(n)
```

Arguments

n integer (order of the matrix)

Details

This function construct Hadamard matrix by multiple of 2 Hadamard matrix. It Returns the Hadamard Matrix, if it is not possible NULL is returned.

Value

Hadamard matrix of order "n"

References

Sylvester, J.J. (1967). Thoughts on orthogonal matrices, simultaneous sign-succession and Tesselated pavements in two or more colours, with applications to Newton's rule, ornamental Tie-work, and the theory of numbers. *Phil. Mag.*,34, 461-475.

Sylvester, J.J. (1968). Problem 2511. *Math. Questions and solutions*, 10, 74.

Hedayat, A. and Wallis, W.D. (1978). Hadamard Matrices and Their Application. *Ann. Stat.*, 6, 1184-1238.

Examples

```

kronecker_matrix(8)
#      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
#[1,]  1   1   1   1   1   1   1   1
#[2,]  1  -1   1  -1   1  -1   1  -1
#[3,]  1   1  -1  -1   1   1  -1  -1
#[4,]  1  -1  -1   1   1  -1  -1   1
#[5,]  1   1   1   1  -1  -1  -1  -1
#[6,]  1  -1   1  -1  -1   1  -1   1
#[7,]  1   1  -1  -1  -1  -1   1   1
#[8,]  1  -1  -1   1  -1   1   1  -1
kronecker_matrix(12)
#NULL

```

kro_method

kro_method

Description

kro_method internal function. Not exported.

Usage

```
kro_method(r)
```

Arguments

r integer (order of the matrix)

Value

$r/2$ or NULL.

References

Sylvester, J.J. (1967). Thoughts on orthogonal matrices, simultaneous sign-succession and Tesselated pavements in two or more colours, with applications to Newton's rule, ornamental Tie-work, and the theory of numbers. *Phil. Mag.*,34, 461-475.

Sylvester, J.J. (1968). Problem 2511. *Math. Questions and solutions*, 10, 74.

Hedayat, A. and Wallis, W.D. (1978). Hadamard Matrices and Their Application. *Ann. Stat.*, 6, 1184-1238.

| | |
|-----------------|------------------------|
| method1_paleyII | <i>method1_paleyII</i> |
|-----------------|------------------------|

Description

method1_paleyII is internal function not exported.

Usage

method1_paleyII(n)

Arguments

n integer

Details

this function checks $q < (n/2) - 1$, q is prime number and $q \equiv 1 \pmod{4}$. if it satisfy it returns q ; otherwise returns NULL.

Value

0 or $(n/2) - 1$

References

Paley, R.E.A.C. (1933). On Orthogonal matrices. J. Combin. Theory, A 57(1), 86-108.

| | |
|-----------------|------------------------|
| method2_paleyII | <i>method2_paleyII</i> |
|-----------------|------------------------|

Description

method2_paleyII is internal function not exported.

Usage

method2_paleyII(n)

Arguments

n integer (order of the matrix)

Details

this function checks $q < (n/4) - 1$, q is prime number and $q \equiv 1 \pmod{4}$. if it satisfy it returns q ; otherwise returns NULL.

Value0 or $(n/4)-1$ **References**

Paley, R.E.A.C. (1933). On Orthogonal matrices. J. Combin. Theory, A 57(1), 86-108.

`miyamotoC``miyamotoC`

Description`miyamotoC`**Usage**`miyamotoC(n)`**Arguments**`n` integer (order of the matrix)**Value**

q matrix

References

Miyamoto, M. (1991). A Construction of Hadamard matrices. J. Math. Phy., 12, 311-320.

Examples

```

miyamotoC(20)
#      [,1] [,2] [,3] [,4] [,5]
#[1,]  0    1    1   -1   -1
#[2,]  1    0   -1   -1    1
#[3,]  1   -1    0    1   -1
#[4,] -1   -1    1    0    1
#[5,] -1    1   -1    1    0

```

| | |
|----------|-----------------|
| nextElem | <i>nextElem</i> |
|----------|-----------------|

Description

nextElem Generate next element of GF.

Usage

nextElem(p1, MF, p, r)

Arguments

| | |
|----|---------|
| p1 | integer |
| MF | integer |
| p | integer |
| r | integer |

Value

A vector of order r, the coefficients of elements.

| | |
|---------|---|
| Normcol | <i>Normcol Normcol performs the Normalisation of column the given matrix.</i> |
|---------|---|

Description

Normcol Normcol performs the Normalisation of column the given matrix.

Usage

Normcol(m)

Arguments

| | |
|---|--------|
| m | Matrix |
|---|--------|

Details

For the given matrix of the first column of the all the -1 elements converting +1 without alter the property of the matrix.

Value

Normalised matrix

Examples

```

PaleyI(8)
#      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
#[1,]   1   1   1   1   1   1   1   1
#[2,]  -1   1  -1  -1   1  -1   1   1
#[3,]  -1   1   1  -1  -1   1  -1   1
#[4,]  -1   1   1   1  -1  -1   1  -1
#[5,]  -1  -1   1   1   1  -1  -1   1
#[6,]  -1   1  -1   1   1   1  -1  -1
#[7,]  -1  -1   1  -1   1   1   1  -1
#[8,]  -1  -1  -1   1  -1   1   1   1
Normcol(PaleyI(8))
#      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
#[1,]   1   1   1   1   1   1   1   1
#[2,]   1  -1   1   1  -1   1  -1  -1
#[3,]   1  -1  -1   1   1  -1   1  -1
#[4,]   1  -1  -1  -1   1   1  -1   1
#[5,]   1   1  -1  -1  -1   1   1  -1
#[6,]   1  -1   1  -1  -1  -1   1   1
#[7,]   1   1  -1   1  -1  -1  -1   1
#[8,]   1   1   1  -1   1  -1  -1  -1

```

Normrow

Normrow Normcol performs the Normalisation of row the given matrix.

Description

Normrow Normcol performs the Normalisation of row the given matrix.

Usage

Normrow(m)

Arguments

m Matrix

Details

For the given matrix of the first row of the all the -1 elements converting +1 without alter the property of the matrix.

Value

Normalised matrix

Examples

```

PaleyII(12)
#      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12]
# [1,]  1   1   1   1   1   1   1  -1  -1  -1  -1  -1
# [2,]  1   1   1  -1  -1   1  -1   1  -1   1   1  -1
# [3,]  1   1   1   1  -1  -1  -1  -1   1   1  -1   1
# [4,]  1  -1   1   1   1  -1  -1   1  -1   1  -1   1
# [5,]  1  -1  -1   1   1   1  -1   1   1  -1   1  -1
# [6,]  1   1  -1  -1   1   1  -1  -1   1   1  -1   1
# [7,]  1  -1  -1  -1  -1  -1  -1  -1  -1  -1  -1  -1
# [8,] -1   1  -1   1   1  -1  -1  -1  -1  -1   1   1
# [9,] -1  -1   1  -1   1   1  -1  -1  -1  -1   1   1
#[10,] -1   1  -1   1  -1   1  -1   1  -1  -1  -1   1
#[11,] -1   1   1  -1   1  -1  -1   1   1  -1  -1  -1
#[12,] -1  -1   1   1  -1   1  -1  -1   1   1  -1  -1
Normrow(PaleyII(12))
#      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12]
# [1,]  1   1   1   1   1   1   1   1   1   1   1   1
# [2,]  1   1   1  -1  -1   1  -1  -1   1  -1  -1   1
# [3,]  1   1   1   1  -1  -1  -1   1  -1   1  -1  -1
# [4,]  1  -1   1   1   1  -1  -1  -1   1  -1   1  -1
# [5,]  1  -1  -1   1   1   1  -1  -1  -1   1  -1   1
# [6,]  1   1  -1  -1   1   1  -1   1  -1  -1   1  -1
# [7,]  1  -1  -1  -1  -1  -1  -1   1   1   1   1   1
# [8,] -1   1  -1   1   1  -1  -1   1   1  -1  -1   1
# [9,] -1  -1   1  -1   1   1  -1   1   1   1  -1  -1
#[10,] -1   1  -1   1  -1   1  -1  -1   1   1   1  -1
#[11,] -1   1   1  -1   1  -1  -1  -1  -1   1   1   1
#[12,] -1  -1   1   1  -1   1  -1   1  -1  -1   1   1

```

PaleyI

*PaleyI***Description**

This function performs constructing the Hadamard matrix by Paley method.

Usage

```
PaleyI(n)
```

Arguments

n integer (order of the matrix)

Details

let $q = n - 1$, and $q \equiv 3 \pmod{4}$, q is the prime number, then obtained the Hadamard matrix of order $q + 1$. if input satisfies these condition it returns Hadamard Matrix; otherwise returns NULL.

Value

hadamard matrix of n

References

Paley, R.E.A.C. (1933). On Orthogonal matrices. J. Combin. Theory, A 57(1), 86-108.

Examples

```
PaleyI(8)
#' PaleyI(8)
#      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
#[1,]  1   1   1   1   1   1   1   1
#[2,] -1   1  -1  -1   1  -1   1   1
#[3,] -1   1   1  -1  -1   1   -1   1
#[4,] -1   1   1   1  -1  -1   1  -1
#[5,] -1  -1   1   1   1  -1  -1   1
#[6,] -1   1  -1   1   1   1  -1  -1
#[7,] -1  -1   1  -1   1   1   1  -1
#[8,] -1  -1  -1   1  -1   1   1   1
PaleyI(16)
#NULL
```

PaleyII

PaleyII

Description

This function create the Hadamard matrix by Paley method 2

Usage

```
PaleyII(n)
```

Arguments

n integer(order of the matrix)

Details

$q=n/2-1$, If there is an Hadamard matrix of order $h>1$, and $q = 1 \pmod{4}$ is a prime number, then there exists an Hadamard matrix of order nh .

Value

Hadamard matrix of order n

References

Paley, R.E.A.C. (1933). On Orthogonal matrices. J. Combin. Theory, A 57(1), 86-108.

Examples

```

PaleyII(12)
#      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12]
# [1,]  1   1   1   1   1   1   1  -1  -1  -1  -1  -1
# [2,]  1   1   1  -1  -1   1  -1   1  -1   1   1  -1
# [3,]  1   1   1   1  -1  -1  -1  -1   1  -1  -1   1   1
# [4,]  1  -1   1   1   1  -1  -1   1  -1   1  -1  -1   1
# [5,]  1  -1  -1   1   1   1  -1   1   1  -1   1  -1  -1
# [6,]  1   1  -1  -1   1   1  -1  -1   1   1  -1  -1   1
# [7,]  1  -1  -1  -1  -1  -1  -1  -1  -1  -1  -1  -1  -1
# [8,] -1   1  -1   1   1  -1  -1  -1  -1  -1   1   1  -1
# [9,] -1  -1   1  -1   1   1  -1  -1  -1  -1  -1   1   1
#[10,] -1   1  -1   1  -1   1  -1   1  -1  -1  -1  -1   1
#[11,] -1   1   1  -1   1  -1  -1   1   1  -1  -1  -1  -1
#[12,] -1  -1   1   1  -1   1  -1  -1   1   1  -1  -1  -1
PaleyII(8)
#NULL

```

PaleyIIPrimePower

PaleyIIPrimePower

Description

PaleyIIPrimePower

Usage

PaleyIIPrimePower(order)

Arguments

order integer

Details

$q=n/2-1$, If there is an Hadamard matrix of order $h>1$, and $q = 1 \pmod{4}$ is a prime power, then there exists an Hadamard matrix of order nh .

Value

Hadamard matrix of the given order.

References

Paley, R.E.A.C. (1933). On Orthogonal matrices. J. Combin. Theory, A 57(1), 86-108.

Examples

```
PaleyIIPrimePower(20)  
PaleyIIPrimePower(24)
```

| | |
|------------------|-------------------------|
| PaleyIPrimePower | <i>PaleyIPrimePower</i> |
|------------------|-------------------------|

Description

PaleyIPrimePower

Usage

```
PaleyIPrimePower(n)
```

Arguments

n integer

Details

let $q = n-1$, and $q = 3 \pmod{4}$, q is the prime power, then obtained the Hadamard matrix of order $q+1$.if input satisfies these condition it returns Hadamard Matrix; otherwise returns NULL.

Value

Hadamard matrix

References

Paley, R.E.A.C. (1933). On Orthogonal matrices. J. Combin. Theory, A 57(1), 86-108.

Examples

```
PaleyIPrimePower(28)  
PaleyIPrimePower(28)  
#NULL
```

ply1

ply1

Description

ply1 -internal function; not exported.

Usage

ply1(q)

Arguments

q integer

Value

Hadamard matrix of order $2(q+1)$

References

Paley, R.E.A.C. (1933). On Orthogonal matrices. J. Combin. Theory, A 57(1), 86-108.

ply2

ply2

Description

ply2 is internal function and not exported

Usage

ply2(q)

Arguments

q integer

Value

Hadamard matrix of order $4(q+1)$

References

Paley, R.E.A.C. (1933). On Orthogonal matrices. J. Combin. Theory, A 57(1), 86-108.

| | |
|-----|------------|
| pow | <i>pow</i> |
|-----|------------|

Description

pow functions finds the exponent of 2.

Usage

```
pow(n)
```

Arguments

n integer

Details

This function checks the given number is the power of 2 or not If the given number is power of 2 it returns the exponent value; otherwise NULL is returned.

Value

power of 2

Examples

```
pow(4)
# 2
pow(5)
#NULL
pow(6)
#NULL
```

| | |
|-------|--------------|
| qhad2 | <i>qhad2</i> |
|-------|--------------|

Description

qhad2 creates the Quadratic residues of the prime number.

Usage

```
qhad2(p)
```

Arguments

p is the integer

Details

The given input is prime number it returns the matrix of order p. if the input is not prime number it returns NULL.

Value

matrix of order p

Examples

```
qhad2(7)
#      [,1] [,2] [,3] [,4] [,5] [,6] [,7]
#[1,]    0   -1   -1    1   -1    1    1
#[2,]    1    0   -1   -1    1   -1    1
#[3,]    1    1    0   -1   -1    1   -1
#[4,]   -1    1    1    0   -1   -1    1
#[5,]    1   -1    1    1    0   -1   -1
#[6,]   -1    1   -1    1    1    0   -1
#[7,]   -1   -1    1   -1    1    1    0
```

QPrimePower

QPrimePower QPrimePower creates the Quadratic residues of the prime number.

Description

QPrimePower QPrimePower creates the Quadratic residues of the prime number.

Usage

QPrimePower(cardin)

Arguments

cardin integer

Details

The given input is prime power it returns the matrix of order cardin. if the input is not prime number then it returns NULL.

Value

matrix of cardin x cardin

Examples

```

QPrimePower(9)
#      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
#[1,]   0   1  -1   1  -1   1  -1   1  -1
#[2,]   1   0   1  -1   1   1  -1  -1  -1
#[3,]  -1   1   0  -1   1  -1  -1   1   1
#[4,]   1  -1  -1   0   1  -1   1   1  -1
#[5,]  -1   1   1   1   0  -1   1  -1  -1
#[6,]   1   1  -1  -1  -1   0   1  -1   1
#[7,]  -1  -1  -1   1   1   1   0  -1   1
#[8,]   1  -1   1   1  -1  -1  -1   0   1
#[9,]  -1  -1   1  -1  -1   1   1   1   0
QPrimePower(36)
#NULL

```

quadprime

quadprime

Description

quadprime is a internal function not exported.

Usage

```
quadprime(p)
```

Arguments

p integer

Details

this function obtain Quadratic residues of GF. It returns squares of odd elements of GF

Value

squares

| | |
|----------------|-----------------------|
| seq_williamson | <i>seq_williamson</i> |
|----------------|-----------------------|

Description

seq_williamson performs the selection of Williamson sequences from dataset

Usage

```
seq_williamson(order)
```

Arguments

| | |
|-------|---------|
| order | integer |
|-------|---------|

Details

Create williamson sequences of given order from the internal dataset williamson_sequences

Williamson sequences are available for length of seq(1,63, 2) except 15, 35, 47, 53, 59 in the internal table.

Value

Required Williamson sequences of order

Source

The Base sequences are obtained The Base sequences were obtained from [Christos Koukouvinos](#) and London (2013).

References

Williamson, J. (1944). Hadamard determinant theorem and the sum of four squares. Duke. Math. J., 11, 65-81.

Williamson, J. (1947). Note on Hadamard's determinant theorem. Bull. Amer. Math. Soc., 53, 608-613.

London, S. 2013. Constructing New Turyn Type Sequences, T-Sequences and Hadamard Matrices. PhD Thesis, University of Illinois at Chicago, Chicago.

See Also

[had_williamson](#) for Williamson construction method using Williamson sequences.

| | |
|-----------|--|
| Turyn_seq | <i>Turyn_seq Turyn_seq performs the selection of the Turyn sequences from dataset. It is internal function not exported.</i> |
|-----------|--|

Description

Turyn_seq Turyn_seq performs the selection of the Turyn sequences from dataset. It is internal function not exported.

Usage

```
Turyn_seq(order)
```

Arguments

order integer

Details

Create Turyn sequences of given order from the internal dataset T_sequences

Turyn type-sequences are available for 28,30,34,36 in the internal table.

Value

Required Turyn sequences of order of x

References

Goethals, J. M. and Seidel, J. J. (1967). Orthogonal matrices with zero diagonal. *Canad. J. Math.*, 19, 259-264.

See Also

[had_goethals_Turyn](#) for Goethals-Seidel construction method using Turyn sequences.

#'

| | |
|------------|--|
| Turyn_to_T | <i>Turyn_to_T</i> internal function. converts Turyn sequences to Base Sequences. |
|------------|--|

Description

Turyn_to_T internal function. converts Turyn sequences to Base Sequences.

Usage

```
Turyn_to_T(dat1, order)
```

Arguments

| | |
|-------|--|
| dat1 | is the Turyn sequences subset exported from Tseq |
| order | integer (order of the matrix) |

Details

dat - Internal dataset containing 4 sequences in long form with length $2n-1$, $2n-1$, n , n . Using the 4 Turyn sequences, the function creates 4 sequences of length $n+p$, $n+p$, n , n . Base Sequences are usually used in creating matrices of Goethel Seidal array.

Turyn type-sequences are available for 28,30,34,36 in the internal table.

Value

Basesequences of length of order

References

Goethals, J. M. and Seidel, J. J. (1967). Orthogonal matrices with zero diagonal. *Canad. J. Math.*, 19, 259-264.

See Also

[had_goethals_Turyn](#) for Goethals-Seidel construction method using Turyn sequences.

| | |
|-------|---|
| T_seq | <i>T_seq T_seq performs the selection of the T sequences from dataset.internal function not exported.</i> |
|-------|---|

Description

T_seq T_seq performs the selection of the T sequences from dataset.internal function not exported.

Usage

T_seq(order)

Arguments

order integer

Details

Create T sequences of given order from the internal dataset T_sequences

T-sequences are available for length of seq(1,73,2) and 83, 101 and 107 in the internal table.

Value

Required Turyn sequences of order of x

Source

The Turyn sequences were obtained from [Christos Koukouvinos](#).

References

Goethals, J. M. and Seidel, J. J. (1967). Orthogonal matrices with zero diagonal. *Canad. J. Math.*, 19, 259-264.

See Also

[had_goethals_T](#) for Goethals-Seidel construction method using T-sequences.

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