

# Package ‘RMTstat’

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**Title** Distributions, Statistics and Tests Derived from Random Matrix Theory

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**Description** Functions for working with the Tracy-Widom laws and other distributions related to the eigenvalues of large Wishart matrices. The tables for computing the Tracy-Widom densities and distribution functions were computed by functions were computed by Momar Dieng's MATLAB package ``RMLab''. This package is part of a collaboration between Iain Johnstone, Zongming Ma, Patrick Perry, and Morteza Shahram.

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## R topics documented:

MarcenkoPastur . . . . .	2
TracyWidom . . . . .	3
WishartMax . . . . .	4
WishartMaxPar . . . . .	6
WishartSpike . . . . .	7
WishartSpikePar . . . . .	8

<b>Index</b>	<b>10</b>
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**Description**

Density, distribution function, quantile function and random generation for the Marčenko-Pastur distribution, the limiting distribution of the empirical spectral measure for a large white Wishart matrix.

**Usage**

```
dmp( x, ndf=NA, pdim=NA, var=1, svr=ndf/pdim, log = FALSE )
pmp( q, ndf=NA, pdim=NA, var=1, svr=ndf/pdim, lower.tail = TRUE, log.p = FALSE )
qmp( p, ndf=NA, pdim=NA, var=1, svr=ndf/pdim, lower.tail = TRUE, log.p = FALSE )
rmp( n, ndf=NA, pdim=NA, var=1, svr=ndf/pdim )
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
<code>ndf</code>	the number of degrees of freedom for the Wishart matrix.
<code>pdim</code>	the number of dimensions (variables) for the Wishart matrix.
<code>var</code>	the population variance.
<code>svr</code>	samples to variables ratio; the number of degrees of freedom per dimension.
<code>log, log.p</code>	logical; if TRUE, probabilities <code>p</code> are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .

**Details**

The concentration can either be given explicitly, or else computed from the given `ndf` and `pdim`. If `var` is not specified, it assumes the default of 1.

The Marčenko-Pastur law is the limit of the random probability measure which puts equal mass on all `pdim` eigenvalues of a normalized `pdim`-dimensional white Wishart matrix with `ndf` degrees of freedom and scale parameter `diag(var, var, ..., var)`. It is assumed that `ndf` goes to infinity, and `ndf/pdim` goes to nonzero constant called the "samples-to-variables ratio" (`svr`).

**Value**

`dmp` gives the density, `pmp` gives the distribution function, `qmp` gives the quantile function, and `rmp` generates random deviates.

**Author(s)**

Iain M. Johnstone, Zongming Ma, Patrick O. Perry and Morteza Shahram

**Source**

Other than the density, these functions are relatively slow and imprecise.

The distribution function is computed with `integrate`. The quantiles are computed via bisection using `uniroot`. Random variates are generated using the inverse CDF.

**References**

Marčenko, V.A. and Pastur, L.A. (1967). Distribution of eigenvalues for some sets of random matrices. *Sbornik: Mathematics* **1**, 457–483.

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TracyWidom

*The Tracy-Widom Distributions*


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**Description**

Density, distribution function, quantile function, and random generation for the Tracy-Widom distribution with order parameter beta.

**Usage**

```
dtw(x, beta=1, log = FALSE)
ptw(q, beta=1, lower.tail = TRUE, log.p = FALSE)
qtw(p, beta=1, lower.tail = TRUE, log.p = FALSE)
rtw(n, beta=1)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
<code>beta</code>	the order parameter (1, 2, or 4).
<code>log, log.p</code>	logical; if TRUE, probabilities p are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .

**Details**

If beta is not specified, it assumes the default value of 1.

The Tracy-Widom law is the edge-scaled limiting distribution of the largest eigenvalue of a random matrix from the  $\beta$ -ensemble. Supported values for beta are 1 (Gaussian Orthogonal Ensemble), 2 (Gaussian Unitary Ensemble), and 4 (Gaussian Symplectic Ensemble).

**Value**

`dtw` gives the density, `ptw` gives the distribution function, `qtw` gives the quantile function, and `rtw` generates random deviates.

**Author(s)**

Iain M. Johnstone, Zongming Ma, Patrick O. Perry and Morteza Shahram

**Source**

The distribution and density functions are computed using a lookup table. They have been pre-computed at 769 values uniformly spaced between  $-10$  and  $6$  using MATLAB's `bvp4c` solver to a minimum accuracy of about  $3.4e-08$ . For all other points, the values are gotten from a cubic Hermite polynomial interpolation. The MATLAB software for computing the grid of values is part of RMLab, a package written by Momar Dieng.

The quantiles are computed via bisection using [uniroot](#).

Random variates are generated using the inverse CDF.

**References**

Dieng, M. (2006). Distribution functions for edge eigenvalues in orthogonal and symplectic ensembles: Painlevé representations. *arXiv:math/0506586v2 [math.PR]*.

Tracy, C.A. and Widom, H. (1994). Level-spacing distributions and the Airy kernel. *Communications in Mathematical Physics* **159**, 151–174.

Tracy, C.A. and Widom, H. (1996). On orthogonal and symplectic matrix ensembles. *Communications in Mathematical Physics* **177**, 727–754.

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WishartMax

*The White Wishart Maximum Eigenvalue Distributions*

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**Description**

Density, distribution function, quantile function, and random generation for the maximum eigenvalue from a white Wishart matrix (sample covariance matrix) with `ndf` degrees of freedom, `pdim` dimensions, population variance `var`, and order parameter `beta`.

**Usage**

```
dWishartMax(x, ndf, pdim, var=1, beta=1, log = FALSE)
pWishartMax(q, ndf, pdim, var=1, beta=1, lower.tail = TRUE, log.p = FALSE)
qWishartMax(p, ndf, pdim, var=1, beta=1, lower.tail = TRUE, log.p = FALSE)
rWishartMax(n, ndf, pdim, var=1, beta=1)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
<code>ndf</code>	the number of degrees of freedom for the Wishart matrix

<code>pdim</code>	the number of dimensions (variables) for the Wishart matrix
<code>var</code>	the population variance.
<code>beta</code>	the order parameter (1 or 2).
<code>log, log.p</code>	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .

### Details

If `beta` is not specified, it assumes the default value of 1. Likewise, `var` assumes a default of 1.

A white Wishart matrix is equal in distribution to  $(1/n)X'X$ , where  $X$  is an  $n \times p$  matrix with elements i.i.d. Normal with mean zero and variance `var`. These functions give the limiting distribution of the largest eigenvalue from the such a matrix when `ndf` and `pdim` both tend to infinity.

Supported values for `beta` are 1 for real data and 2 for complex data.

### Value

`dWishartMax` gives the density, `pWishartMax` gives the distribution function, `qWishartMax` gives the quantile function, and `rWishartMax` generates random deviates.

### Author(s)

Iain M. Johnstone, Zongming Ma, Patrick O. Perry and Morteza Shahram

### Source

The functions are calculated by applying the appropriate centering and scaling (determined by [WishartMaxPar](#)), and then calling the corresponding functions for the [TracyWidom](#) distribution.

### References

- Johansson, K. (2000). Shape fluctuations and random matrices. *Communications in Mathematical Physics*. **209** 437–476.
- Johnstone, I.M. (2001). On the ditribution of the largest eigenvalue in principal component analysis. *Annals of Statistics*. **29** 295–327.

### See Also

[WishartMaxPar](#), [WishartSpike](#), [TracyWidom](#)

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WishartMaxPar

*White Wishart Maximum Eigenvalue Centering and Scaling*


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### Description

Centering and scaling for the maximum eigenvalue from a white Wishart matrix (sample covariance matrix) with `ndf` degrees of freedom, `pdim` dimensions, population variance `var`, and order parameter `beta`.

### Usage

```
WishartMaxPar(ndf, pdim, var=1, beta=1)
```

### Arguments

<code>ndf</code>	the number of degrees of freedom for the Wishart matrix.
<code>pdim</code>	the number of dimensions (variables) for the Wishart matrix.
<code>var</code>	the population variance.
<code>beta</code>	the order parameter (1 or 2).

### Details

If `beta` is not specified, it assumes the default value of 1. Likewise, `var` assumes a default of 1.

The returned values give appropriate centering and scaling for the largest eigenvalue from a white Wishart matrix so that the centered and scaled quantity converges in distribution to a Tracy-Widom random variable. We use the second-order accurate versions of the centering and scaling given in the references below.

### Value

<code>centering</code>	gives the centering.
<code>scaling</code>	gives the scaling.

### Author(s)

Iain M. Johnstone, Zongming Ma, Patrick O. Perry and Morteza Shahram

### References

El Karoui, N. (2006). A rate of convergence result for the largest eigenvalue of complex white Wishart matrices. *Annals of Probability* **34**, 2077–2117.

Ma, Z. (2008). Accuracy of the Tracy-Widom limit for the largest eigenvalue in white Wishart matrices. *arXiv:0810.1329v1 [math.ST]*.

### See Also

[WishartMax](#), [TracyWidom](#)

**Description**

Density, distribution function, quantile function, and random generation for the maximum eigenvalue from a spiked Wishart matrix (sample covariance matrix) with `ndf` degrees of freedom, `pdim` dimensions, and population covariance matrix `diag(spike+var, var, var, . . . , var)`.

**Usage**

```
dWishartSpike(x, spike, ndf=NA, pdim=NA, var=1, beta=1, log = FALSE)
pWishartSpike(q, spike, ndf=NA, pdim=NA, var=1, beta=1, lower.tail = TRUE, log.p = FALSE)
qWishartSpike(p, spike, ndf=NA, pdim=NA, var=1, beta=1, lower.tail = TRUE, log.p = FALSE)
rWishartSpike(n, spike, ndf=NA, pdim=NA, var=1, beta=1)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
<code>spike</code>	the value of the spike.
<code>ndf</code>	the number of degrees of freedom for the Wishart matrix.
<code>pdim</code>	the number of dimensions (variables) for the Wishart matrix.
<code>var</code>	the population (noise) variance.
<code>beta</code>	the order parameter (1 or 2).
<code>log, log.p</code>	logical; if TRUE, probabilities <code>p</code> are given as <code>log(p)</code> .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .

**Details**

The spiked Wishart is a random sample covariance matrix from multivariate normal data with `ndf` observations in `pdim` dimensions. The spiked Wishart has one population covariance eigenvalue equal to `spike+var` and the rest equal to `var`. These functions are related to the limiting distribution of the largest eigenvalue from such a matrix when `ndf` and `pdim` both tending to infinity, with their ratio tending to a nonzero constant.

For the spiked distribution to exist, `spike` must be greater than `sqrt(pdim/ndf)*var`.

Supported values for `beta` are 1 for real data and 2 for complex data.

**Value**

`dWishartSpike` gives the density, `pWishartSpike` gives the distribution function, `qWishartSpike` gives the quantile function, and `rWishartSpike` generates random deviates.

**Author(s)**

Iain M. Johnstone, Zongming Ma, Patrick O. Perry and Morteza Shahram

**References**

- Baik, J., Ben Arous, G., and Pécché, S. (2005). Phase transition of the largest eigenvalue for non-null complex sample covariance matrices. *Annals of Probability* **33**, 1643–1697.
- Baik, J. and Silverstein, J. W. (2006). Eigenvalues of large sample covariance matrices of spiked population models. *Journal of Multivariate Analysis* **97**, 1382-1408.
- Paul, D. (2007). Asymptotics of sample eigenstructure for a large dimensional spiked covariance model. *Statistica Sinica*. **17**, 1617–1642.

**See Also**

[WishartSpikePar](#), [WishartMax](#)

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WishartSpikePar

*Spiked Wishart Eigenvalue Centering and Scaling*

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**Description**

Centering and scaling for the sample eigenvalue from a spiked Wishart matrix (sample covariance matrix) with `ndf` degrees of freedom, `pdim` dimensions, and population covariance matrix `diag(spike+var, var, var, . . . , var)`.

**Usage**

```
WishartSpikePar( spike, ndf=NA, pdim=NA, var=1, beta=1 )
```

**Arguments**

<code>spike</code>	the value of the spike.
<code>ndf</code>	the number of degrees of freedom for the Wishart matrix.
<code>pdim</code>	the number of dimensions (variables) for the Wishart matrix.
<code>var</code>	the population (noise) variance.
<code>beta</code>	the order parameter (1 or 2).

**Details**

The returned values give appropriate centering and scaling for the largest eigenvalue from a spiked Wishart matrix so that the centered and scaled quantity converges in distribution to a normal random variable with mean 0 and variance 1.

For the spiked distribution to exist, `spike` must be greater than  $\sqrt{\text{pdim}/\text{ndf}} * \text{var}$ .

Supported values for `beta` are 1 for real data and 2 for complex data.



**Value**

centering        gives the centering.  
scaleing        gives the scaling.

**Author(s)**

Iain M. Johnstone, Zongming Ma, Patrick O. Perry and Morteza Shahram

**References**

Baik, J., Ben Arous, G., and Pécché, S. (2005). Phase transition of the largest eigenvalue for non-null complex sample covariance matrices. *Annals of Probability* **33**, 1643–1697.

Baik, J. and Silverstein, J. W. (2006). Eigenvalues of large sample covariance matrices of spiked population models. *Journal of Multivariate Analysis* **97**, 1382-1408.

Paul, D. (2007). Asymptotics of sample eigenstructure for a large dimensional spiked covariance model. *Statistica Sinica* **17**, 1617–1642.

**See Also**

[WishartSpike](#)

# Index

## \* **distribution**

- MarcenkoPastur, [2](#)
- TracyWidom, [3](#)
- WishartMax, [4](#)
- WishartMaxPar, [6](#)
- WishartSpike, [7](#)
- WishartSpikePar, [8](#)

dmp (MarcenkoPastur), [2](#)  
dtw (TracyWidom), [3](#)  
dWishartMax (WishartMax), [4](#)  
dWishartSpike (WishartSpike), [7](#)

integrate, [3](#)

MarcenkoPastur, [2](#)

pmp (MarcenkoPastur), [2](#)  
ptw (TracyWidom), [3](#)  
pWishartMax (WishartMax), [4](#)  
pWishartSpike (WishartSpike), [7](#)

qmp (MarcenkoPastur), [2](#)  
qtw (TracyWidom), [3](#)  
qWishartMax (WishartMax), [4](#)  
qWishartSpike (WishartSpike), [7](#)

rmp (MarcenkoPastur), [2](#)  
rtw (TracyWidom), [3](#)  
rWishartMax (WishartMax), [4](#)  
rWishartSpike (WishartSpike), [7](#)

TracyWidom, [3](#), [5](#), [6](#)

uniroot, [3](#), [4](#)

WishartMax, [4](#), [6](#), [8](#)  
WishartMaxPar, [5](#), [6](#)  
WishartSpike, [5](#), [7](#), [9](#)  
WishartSpikePar, [8](#), [8](#)