

# Package ‘cpr’

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**Title** Control Polygon Reduction

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**License** GPL (>= 2)

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cpr-package

*cpr: Control Polygon Reduction*


---

## Description

The cpr package implements the control polygon reduction and control net reduction methods for finding parsimonious B-spline regression models.

This package was part of Peter DeWitt's Ph.D. dissertation which was overseen by his advisors Samantha MaWhinney and Nichole Carlson.

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bsplineD                      *B-spline Derivatives*

---

### Description

Generate the first and second derivatives of a B-spline Basis.

### Usage

```
bsplineD(x, iknots = NULL, df = NULL, bknots = range(x), order = 4L,  
         derivative = 1L)
```

### Arguments

x	a numeric vector
iknots	internal knots
df	degrees of freedom: sum of the order and internal knots. Ignored if iknots is specified.
bknots	boundary knot locations, defaults to range(x).
order	order of the piecewise polynomials, defaults to 4L.
derivative,	(integer) first or second derivative

### References

C. de Boor, "A practical guide to splines. Revised Edition," Springer, 2001.  
H. Prautzsch, W. Boehm, M. Paluszny, "Bezier and B-spline Techniques," Springer, 2002.

### See Also

[bsplines](#)

### Examples

```
set.seed(42)  
  
xvec <- seq(0.1, 9.9, length = 1000)  
iknots <- sort(runif(rpois(1, 3), 1, 9))  
bknots <- c(0, 10)  
  
# basis matrix and the first and second derivatives thereof, for cubic (order =  
# 4) b-splines  
bmat <- bsplines(xvec, iknots, bknots = bknots)  
bmat1 <- bsplineD(xvec, iknots, bknots = bknots, derivative = 1)  
bmat2 <- bsplineD(xvec, iknots, bknots = bknots, derivative = 2)  
  
# control polygon ordinates
```

```

theta <- runif(length(iknots) + 4L, -5, 5)

# plot data
plot_data <-
  dplyr::data_frame(x = xvec,
                    Spline = as.numeric(bmat %% theta),
                    "First Derivative" = as.numeric(bmat1 %% theta),
                    "Second Derivative" = as.numeric(bmat2 %% theta))
plot_data <- tidyr::gather(plot_data, key = key, value = value, -x)

ggplot2::ggplot(plot_data) +
  ggplot2::aes(x = x, y = value, color = key) +
  ggplot2::geom_line() +
  ggplot2::geom_hline(yintercept = 0) +
  ggplot2::geom_vline(xintercept = iknots, linetype = 3)

```

---

 bsplines

*B-Splines*


---

### Description

An implementation of Carl de Boor's recursive algorithm for building B-splines.

### Usage

```
bsplines(x, iknots = NULL, df = NULL, bknots = range(x), order = 4L)
```

### Arguments

x	a numeric vector
iknots	internal knots
df	degrees of freedom: sum of the order and internal knots. Ignored if iknots is specified.
bknots	boundary knot locations, defaults to range(x).
order	order of the piecewise polynomials, defaults to 4L.

### Details

The difference between this function and `splines::bs` come in the attributes associated with the output and default options. The `cpr::bsplines` call is intended to simplify the work needed with respect to the control polygon reduction. Further, the implementation of `cpr::bsplines` is in C++ and tends to be faster than `splines::bs`.

See the vignette("cpr-pkg", package = "cpr") for a detailed comparison between the `bsplines` and `bs` calls.

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

**References**

C. de Boor, "A practical guide to splines. Revised Edition," Springer, 2001.

H. Prautzsch, W. Boehm, M. Paluszny, "Bezier and B-spline Techniques," Springer, 2002.

**See Also**

[plot.cpr\\_bs](#) for plotting the basis, [bsplineD](#) for building the basis matrices for the first and second derivative of a B-spline.

See [update\\_bsplines](#) for info on a tool for updating a `cpr_bs` object. This is a similar method to the [update](#) function from the `stats` package.

**Examples**

```
# build a vector of values to transform
xvec <- seq(-3, 5, length = 100)

# cubic b-spline
bmat <- bsplines(xvec, iknots = c(-2, 0, 0.2))
bmat

# plot the splines
plot(bmat)

# If you want a second x-axis to show the x-values try the following:
second_x_axis <- round(stats::quantile(xvec, probs = seq(0, 1, by = .2)), 2)

plot(bmat) +
  ggplot2::annotate(geom = "text", x = second_x_axis, y = -0.02, label = second_x_axis) +
  ggplot2::annotate(geom = "linrange", x = second_x_axis, ymin = -0.05, ymax = -0.04) +
  ggplot2::coord_cartesian(ylim = c(0, 1))

# quadratic splines
bmat <- bsplines(xvec, iknots = c(-2, 0, 0.2), order = 3L)
bmat
plot(bmat) + ggplot2::ggtitle("Quadratic B-splines")
```

---

 btensor

*btensor*


---

**Description**

Tensor products of B-splines.

**Usage**

```
btensor(x, df = NULL, iknots = NULL, bknots, order)
```

**Arguments**

<code>x</code>	a list of variables to build B-spline transforms of. The tensor product of these B-splines will be returned.
<code>df</code>	degrees of freedom. a list of the degrees of freedom for each marginal.
<code>iknots</code>	a list of internal knots for each <code>x</code> . If omitted, the default is to place no internal knots for all <code>x</code> . If specified, the list needs to contain the internal knots for all <code>x</code> . If <code>df</code> and <code>iknots</code> are both given, the <code>df</code> will take precedence.
<code>bknots</code>	a list of boundary knots for each <code>x</code> . As with the <code>iknots</code> , if omitted the default will be to use the range of each <code>x</code> . If specified, the use must specify the <code>bknots</code> for each <code>x</code> .
<code>order</code>	a list of the order for each <code>x</code> ; defaults to 4L for all <code>x</code> .

**Details**

The return form this function is the tensor product of the B-splines transformations for the given variables. Say we have variables X, Y, and Z to build the tensor product of. The columns of the returned matrix correspond to the column products of the three B-splines:

```
x1y1z1 x2y1z1 x3y1z1 x4y1z1 x1y2z1 x2y2z1 ... x4y4z4
```

for three fourth order B-splines with no internal knots. The columns of X cycle the quickest, followed by Y, and then Z. This would be the same result as `model.matrix( ~ bsplines(X) : bsplines(Y) : bsplines(Z)`

See `vignette("cpr-pkg", package = "cpr")` for more details.

**Value**

A matrix with a class `cpr_bt`

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

**Examples**

```
tp <- with(mtcars,
  btensor(x = list(displ, hp, mpg),
    iknots = list(numeric(0), c(100, 150), numeric(0)))
)
tp
utils::str(tp)

# The equivalent matrix is could be generated as follows
tp2 <- model.matrix( ~ splines::bs(displ, intercept = TRUE) :
  splines::bs(hp, knots = c(100, 150), intercept = TRUE) :
  splines::bs(mpg, intercept = TRUE) + 0,
  data = mtcars)
```

```
all.equal(tp2, unclass(tp), check.attributes = FALSE)
```

---

build\_tensor

*Build Tensor*

---

## Description

Tensor products of Matrices.

## Usage

```
build_tensor(x, ...)
```

## Arguments

x                    a matrix, or list of numeric matrices, build the tensor product  
...                   additional numeric matrices to build the tensor product

## Value

A matrix

## Author(s)

Peter DeWitt <dewittpe@gmail.com>

## Examples

```
# see vignette("cpr-pkg", package = "cpr") for details on tensor products.  
  
A <- matrix(1:4, nrow = 10, ncol = 20)  
B <- matrix(1:6, nrow = 10, ncol = 6)  
  
# Two ways of building the same tensor product  
tensor1 <- build_tensor(A, B)  
tensor2 <- build_tensor(list(A, B))  
all.equal(tensor1, tensor2)  
  
# a three matrix tensor product  
tensor3 <- build_tensor(A, B, B)  
str(tensor3)
```

---

 cn *Control Nets*


---

**Description**

Generate the control net for a uni-variable B-spline

**Usage**

```

cn(x, ...)

## S3 method for class 'cpr_bt'
cn(x, theta, ...)

## S3 method for class 'formula'
cn(formula, data, method = stats::lm, ...,
    keep_fit = FALSE, check_rank = TRUE)

## S3 method for class 'cpr_cn'
print(x, ...)

## S3 method for class 'cpr_cn'
summary(object, ...)

```

**Arguments**

x	a cpr_bs object
...	arguments passed to the regression method
theta	a vector of (regression) coefficients, the ordinates of the control net.
formula	a formula that is appropriate for regression method being used.
data	a required data.frame
method	the regression method such as <a href="#">lm</a> , <a href="#">glm</a> , <a href="#">lmer</a> , <a href="#">geeglm</a> , ...
keep_fit	(logical, defaults to FALSE). If TRUE the regression model fit is retained and returned in the the fit element. If FALSE the regression model is not saved and the fit element will be NA.
check_rank	(logical, defaults to TRUE) if TRUE check that the design matrix is full rank.
object	a cpr_cn object

**Details**

cn generates the control net for the given B-spline function. There are several methods for building a control net.



**Value**

a `cpr_cnr` object. This is a list with the following elements. Some of the elements are omitted when using the `cn.cpr_bt` method.

**cn** the control net, `data.frame` with each row defining a vertex of the control net

**bspline\_list** A list of the marginal B-splines

**call** the call

**keep\_fit** logical, indicates if the regression models was retained

**fit** if `isTRUE(keep_fit)` then the regression model is here, else NA.

**coefficients** regression coefficients, only the fixed effects if a mixed effects model was used.

**vcov** The variance-covariance matrix for the coefficients

**loglik** The log-likelihood for the regression model

**rmse** The root mean squared error for the regression models

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

---

 cnr

*Control Net Reduction*


---

**Description**

Run the Control Net Reduction Algorithm.

**Usage**

```
cnr(x, keep = -1, p = 2, margin, n_polycoef = 20L,
    progress = interactive(), ...)
```

```
## S3 method for class 'cpr_cnr'
summary(object, ...)
```

**Arguments**

<code>x</code>	a <code>cnr_cp</code> or <code>cnr_tensor</code> object
<code>keep</code>	keep (store) the regression fit for the first <code>keep</code> <code>cpr_cnr</code> objects in the list returned by <code>cnr</code> .
<code>p</code>	defaults to <code>2L</code> , the $L^p$ norm used in determining the influence weight of each internal knot. Passed to <a href="#">influence_weights</a> .
<code>margin</code>	the margins to apply the CNR algorithm to. Passed to <a href="#">influence_weights</a> .
<code>n_polycoef</code>	the number of polynomial coefficients to use when assessing the influence of each internal knot.
<code>progress</code>	show a progress bar.
<code>...</code>	not currently used
<code>object</code>	a <code>cpr_cnr</code> object

**Details**

cnr runs the control net reduction algorithm.

keep will keep the regression fit as part of the cnr\_cp object for models with up to and including keep fits. For example, if keep = 10 then the resulting cnr\_cnr object will have the regression fit stored in the first keep + 1 (zero internal knots, one internal knot, ..., keep internal knots) cnr\_cp objects in the list. The limit on the number of stored regression fits is to keep memory usage down.

**Author(s)**

Peter DeWitt <dewitttpe@gmail.com>

**See Also**

[influence\\_weights](#), [cpr](#) for the uni-variable version, Control Polygon Reduction.

---

 cp

*Control Polygons*


---

**Description**

Generate the control polygon for a uni-variable B-spline

**Usage**

```
cp(x, ...)

## S3 method for class 'cpr_bs'
cp(x, theta, ...)

## S3 method for class 'formula'
cp(formula, data, method = stats::lm, ...,
    keep_fit = FALSE, check_rank = TRUE)

## S3 method for class 'cpr_cp'
print(x, ...)

## S3 method for class 'cpr_cp'
summary(object, wiggle = FALSE, integrate.args = list(),
    ...)

## S3 method for class 'cpr_cp'
plot(x, ..., show_cp = TRUE, show_spline = FALSE,
    show_xi = TRUE, color = FALSE, n = 100)
```

**Arguments**

x	a cpr_bs object
...	arguments passed to the regression method
theta	a vector of (regression) coefficients, the ordinates of the control polygon.
formula	a formula that is appropriate for regression method being used.
data	a required data.frame
method	the regression method such as <code>lm</code> , <code>glm</code> , <code>lmer</code> , <code>geeglm</code> , ...
keep_fit	(logical, default value is FALSE). If TRUE the regression model fit is retained and returned in as the <code>fit</code> element. If FALSE the <code>fit</code> element will be NA.
check_rank	(logical, defaults to TRUE) if TRUE check that the design matrix is full rank.
object	a cpr_cp object
wiggle	logical, if TRUE then the integral of the squared second derivative of the spline function will be calculated via <code>stats::integrate</code> .
integrate.args	a list of arguments passed to <code>cpr::wiggle</code> and ultimately <code>stats::integrate</code> .
show_cp	logical (default TRUE), show the control polygon(s)?
show_spline	logical (default FALSE) to plot the spline function?
show_xi	logical (default TRUE) use <code>geom_rug</code> to show the location of the knots in the respective control polygons.
color	Boolean (default FALSE) if more than one cpr_cp object is to be plotted, set this value to TRUE to have the graphic in color (linetypes will be used regardless of the color setting).
n	the number of data points to use for plotting the spline

**Details**

cp generates the control polygon for the given B-spline function.

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

**Examples**

```
# Support
xvec <- seq(0, 6, length = 500)

# Define the basis matrix
bmat1 <- cpr::bsplines(x = xvec, iknots = c(1, 1.5, 2.3, 4, 4.5))
bmat2 <- cpr::bsplines(x = xvec)

# Define the control vertices ordinates
theta1 <- c(1, 0, 3.5, 4.2, 3.7, -0.5, -0.7, 2, 1.5)
theta2 <- c(1, 3.4, -2, 1.7)
```

```

# build the two control polygons
cp1 <- cp(bmat1, theta1)
cp2 <- cp(bmat2, theta2)

# black and white plot
plot(cp1)
plot(cp1, show_spline = TRUE)

# multiple control polygons
plot(cp1, cp2, show_spline = TRUE)
plot(cp1, cp2, color = TRUE)
plot(cp1, cp2, show_spline = TRUE, color = TRUE)

# via formula
dat <- dplyr::data_frame(x = xvec, y = sin((x - 2)/pi) + 1.4 * cos(x/pi))
cp3 <- cp(y ~ cpr::bsplines(x) + 0, data = dat)

# plot the control polygon, spline and target data.
plot(cp3, show_spline = TRUE) +
  ggplot2::geom_line(mapping = ggplot2::aes_string(x = "x", y = "y"),
                    data = dat, linetype = 2, color = "red")

```

---

cpr

*Control Polygon Reduction*


---

## Description

Run the Control Polygon Reduction Algorithm.

## Usage

```
cpr(x, keep = -1, p = 2, progress = interactive(), ...)
```

```
## S3 method for class 'cpr_cpr'
summary(object, ...)
```

## Arguments

x	a cpr_cp object
keep	keep (store) the regression fit for models with keep or fewer internal knots, e.g., keep = 3 will result in the regression fit for models with 0, 1, 2, and 3 internal knots being saved in their respective cpr_cp objects. The default is keep = -1 so that no regression models are retained.
p	defaults to 2L, the $L^p$ norm used in determining the influence weight of each internal knot.
progress	show a progress bar.
...	not currently used
object	a cpr_cpr object

**Details**

cpr runs the control polygon reduction algorithm.

keep will keep the regression fit as part of the cpr\_cp object for models with up to and including keep fits. For example, if keep = 10 then the resulting cpr\_cpr object will have the regression fit stored in the first keep + 1 (zero internal knots, one internal knot, ..., keep internal knots) cpr\_cp objects in the list. The limit on the number of stored regression fits is to keep memory usage down.

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

---

cp\_value                      *Control Polygon Diagnostics*

---

**Description**

Collection of functions for inspection and analysis of the control polygons

**Usage**

```
cp_value(obj, x)
```

```
cp_diff(cp1, cp2)
```

**Arguments**

obj	a cpr_cp object or data.frame where the first column is the abscissa and the second column is the ordinate for the control polygon vertices.
x	abscissa at which to determine the ordinate on control polygon cp
cp1	a cpr_cp object
cp2	a cpr_cp object

**Value**

cp\_value returns the ordinate on the control polygon line segment for the abscissae x given. x could be a control vertex or on a line segment defined by two control vertices of the control polygon provided.

cp\_diff returns the absolute vertical distance between the control vertices of cp1 to the control polygon cp2.

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

extract\_cpr\_bsplines *Extract the bspline or btensor call from a formula*

---

**Description**

Non-exported function. Might be depericated.

**Usage**

```
extract_cpr_bsplines(form)
```

**Arguments**

form            a formula

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

---

generate\_cp\_formula\_data  
*Generate Control Polygon Formula and Data*

---

**Description**

Construct a `data.frame` and formula to be passed to the regression modeling tool to generate a control polygon.

**Usage**

```
generate_cp_formula_data(f, .data)
```

**Arguments**

f                a formula  
.data            the data set containing the variables in the formula

**Details**

This function is expected to be called from within the `cpr::cp` function and is not expected to be called by the end user directly.

`generate_cp_data` exists because of the need to build what could be considered a varying means model.  $y \sim \text{bsplines}(x1) + x2$  will generate a rank deficient model matrix—the rows of the bspline basis matrix sum to one with is perfectly collinear with the implicit intercept term. Specifying a formula  $y \sim \text{bsplines}(x1) + x2 - 1$  would work if `x2` is a continuous variable. If, however, `x2` is a factor, or coerced to a factor, then the model matrix will again be rank deficient as a column for all levels of the factor will be generated. We need to replace the intercept column of the model matrix with the bspline. This also needs to be done for a variety of possible model calls, `lm`, `lmer`, etc.

By returning an explicit formula and `data.frame` for use in the fit, we hope to reduce memory use and increase the speed of the `cpr` method.

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

---

get\_spline

*Get the Control Polygon and the Spline Function*

---

**Description**

Generate data.frames for interpolating and plotting a spline function, given a `cpr_cp` or `cpr_cn` object.

**Usage**

```
get_spline(x, margin = 1, at, n = 100)
```

**Arguments**

- |                     |                                                                                                                                                                                                                                                                                                                                                          |
|---------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>      | a <code>cpr_cp</code> or <code>cpr_cn</code> object.                                                                                                                                                                                                                                                                                                     |
| <code>margin</code> | an integer identifying the marginal of the control net to slice along. Only used when working <code>x</code> is a <code>cpr_cn</code> object.                                                                                                                                                                                                            |
| <code>at</code>     | point value for marginals not defined in the <code>margin</code> . Only used when <code>x</code> is a <code>cpr_cn</code> object. Expected input is a list of length <code>length(attr(x, "bspline_list"))</code> . Entries for elements <code>margin</code> are ignored. If omitted, the midpoint between the boundary knots for each marginal is used. |
| <code>n</code>      | the length of sequence to use for interpolating the spline function.                                                                                                                                                                                                                                                                                     |

**Details**

A control polygon, `cpr_cp` object, has a spline function  $f(x)$ . `get_spline` returns a list of two `data.frame`. The `cp` element is a `data.frame` with the  $(x, y)$  coordinates control points and the `spline` element is a `data.frame` with  $n$  rows for interpolating  $f(x)$ .

For a control net, `cpr_cn` object, the return is the same as for a `cpr_cp` object, but conceptually different. Where a `cpr_cp` objects have a uni-variable spline function, `cpr_cn` have multi-variable spline surfaces. `get_spline` returns a "slice" of the higher dimensional object. For example, consider a three-dimensional control net defined on the unit cube with marginals  $x_1, x_2$ , and  $x_3$ . The implied spline surface is the function  $f(x_1, x_2, x_3)$ . `get_spline(x, margin = 2, at = list(0.2, NA, 0.5))` would return the control polygon and spline surface for  $f(0.2, x, 0.5)$ .

See [get\\_surface](#) for taking a two-dimensional slice of a three-plus dimensional control net, or, for generating a useful data set for plotting the surface of a two-dimensional control net.

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

**See Also**

[get\\_surface](#)

**Examples**

```
data(spdg, package = "cpr")

## Extract the control polygon and spline for plotting. We'll use base R
## graphics for this example.
a_cp <- cp(pdg ~ bsplines(day, df = 10), data = spdg)

cp_and_spline <- get_spline(a_cp)
plot(cp_and_spline$cp, type = "b")
points(cp_and_spline$spline, type = "l")
grid()

# compare to the cpr:::plot.cpr_cp method
plot(a_cp, show_spline = TRUE)
```

---

get\_surface

*Get Two-Dimensional Control Net and Surface from n-dimensional Control Nets*

---

**Description**

Get Two-Dimensional Control Net and Surface from n-dimensional Control Nets



**Usage**

```
get_surface(x, margin = 1:2, at, n = 100)
```

**Arguments**

x	a cpr_cn object
margin	an integer identifying the marginal of the control net to slice along. Only used when working x is a cpr_cn object.
at	point value for marginals not defined in the margin. Only used when x is a cpr_cn object. Expected input is a list of length length(attr(x, "bspline_list")). Entries for elements marginal are ignored. If omitted, the midpoint between the boundary knots for each marginal is used.
n	the length of sequence to use for interpolating the spline function.

**See Also**

[get\\_spline](#)

**Examples**

```
## Extract the control net and surface from a cpr_cn object.
a_cn <- cn(pdg ~ btensor(list(day, age), df = list(15, 3), order = list(3, 2)),
          data = spdg)

cn_and_surface <- get_surface(a_cn, n = 50)
str(cn_and_surface, max.level = 2)

par(mfrow = c(1, 2))
with(cn_and_surface$cn,
     plot3D::persp3D(unique(Var1),
                    unique(Var2),
                    matrix(z,
                          nrow = dplyr::n_distinct(Var1),
                          ncol = dplyr::n_distinct(Var2)),
                    main = "Control Net")
)
with(cn_and_surface$surface,
     plot3D::persp3D(unique(Var1),
                    unique(Var2),
                    matrix(z,
                          nrow = dplyr::n_distinct(Var1),
                          ncol = dplyr::n_distinct(Var2)),
                    main = "Surface")
)
```

---

`influence_of`*Influence Of Internal Knots*

---

### Description

Given a control polygon and set of indices, return the influence weight for the specified internal knots and generate graphics showing the original, coarsened, and approximated control polygons.

### Usage

```
influence_of(x, indices, ...)
```

### Arguments

<code>x</code>	a <code>cpr_cp</code> object
<code>indices</code>	an integer vector specifying the elements of <code>attr(x, "knots")</code> to assess. Defaults to all internal knots.
<code>...</code>	Additional arguments passed to <a href="#">influence_weights</a> .

### Value

A `cpr_influence_of` object. This is a list with the following elements:

**weight** A tibble (data.frame) showing the influence weight and relative rank of each internal knot

**orig\_cp** The original control polygon

**indices** The indices of the internal knots assessed.

**coarsened\_cp** A list of control polygons. These are the control polygons built on the coarsened knot sequence

**reinserted\_cp** A list of control polygons. These are the control polygons resulting from the reinsertion of the assessed internal knot.

### Author(s)

Peter DeWitt <dewittpe@gmail.com>

### See Also

[plot.cpr\\_influence\\_of](#), [cp](#)

---

influence_weights	<i>Influence Weights</i>
-------------------	--------------------------

---

**Description**

Determine the influence weight of each internal knot on each marginal of a tensor product.

**Usage**

```
influence_weights(x, p = 2, margin = seq_along(x$bspline_list),
  n_polycoef = 20L)
```

**Arguments**

x	a cpr_cp or cpr_cn object
p	the order of the norm, default p = 2.
margin	The margins to apply CNR to. Ignored for cpr_cp objects.
n_polycoef	for influence weights in tensor products, this parameter set the number of polynomial coefficients to use in each of the marginal calculations. Ignored for cpr_cp objects.

**Value**

A data\_frame with two elements, the internal knots (iknots) and the weights.

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

---

is.cpr_bs	<i>Tests for Objects</i>
-----------	--------------------------

---

**Description**

Test if an R object is of a class specific to the cpr package

**Usage**

```
is.cpr_bs(x)
```

```
is.cpr_bt(x)
```

```
is.cpr_cp(x)
```

```
is.cpr_cpr(x)
```

```
is.cpr_cn(x)
```

```
is.cpr_cnr(x)
```

**Arguments**

x                    an R object

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

---

loglikelihood                    *Determine the (quasi) Log Likelihood for a regression object.*

---

**Description**

Return, via stats::logLik or a custom S3 method, the (quasi) log likelihood of a regression object.

**Usage**

```
loglikelihood(x, ...)
```

**Arguments**

x                    a regression fit object  
...                    passed through to stats::logLik

**Details**

This function is used by cpr::cpr and cpr::cnr to determine the (quasi) log likelihood returned in the cpr\_cpr and cpr\_cnr objects.

Generally this function defaults to stats::logLik. Therefore, if an S3 method for determining the (quasi) log likelihood exists in the workspace everything should work. If an S3 method does not exist you should define one.

See methods(loglikelihood) for a list of the provided methods. The default method uses stats::logLik.

**Value**

the numeric value of the (quasi) log likelihood.

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

**See Also**

[cpr](#) [cnr](#) [logLik](#)

---

matrix_rank	<i>Rank of a Matrix</i>
-------------	-------------------------

---

**Description**

Determine the rank (number of linearly independent columns) of a matrix.

**Usage**

```
matrix_rank(x)
```

**Arguments**

x                    a numeric matrix

**Details**

Implementation via the Armadillo C++ linear algebra library. The function returns the rank of the matrix `x`. The computation is based on the singular value decomposition of the matrix; a `std::runtime_error` exception will be thrown if the decomposition fails. Any singular values less than the tolerance are treated as zeros. The tolerance is  $\max(m, n) * \max\_sv * \text{datum}::\text{eps}$ , where `m` is the number of rows of `x`, `n` is the number of columns of `x`, `max_sv` is the maximal singular value of `x`, and `datum::eps` is the difference between 1 and the least value greater than 1 that is representable.

**Value**

the rank of the matrix as a numeric value.

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

**References**

Conrad Sanderson and Ryan Curtin. Armadillo: a template-based C++ library for linear algebra. Journal of Open Source Software, Vol. 1, pp. 26, 2016.

**Examples**

```
# Check the rank of a matrix
mat <- matrix(rnorm(25000 * 120), nrow = 25000)
Matrix::rankMatrix(mat)[1]
matrix_rank(mat)

# A full rank B-spline basis
bmat <- bsplines(seq(0, 1, length = 100), df = 15)
matrix_rank(bmat)
```

```
# A rank deficient B-spline basis
bmat <- bsplines(seq(0, 1, length = 100), iknots = c(0.001, 0.002))
ncol(bmat)
matrix_rank(bmat)
```

---

plot.cpr\_bs

*Plot B-splines*


---

### Description

Plot B-splines

### Usage

```
## S3 method for class 'cpr_bs'
plot(x, ..., show_xi = TRUE, show_x = FALSE,
     color = TRUE, digits = 2, n = 100)
```

### Arguments

x	a cpr_bs object
show_xi	logical, show the knot locations, using the Greek letter xi, on the x-axis
show_x	logical, show the x values of the knots on the x-axis
color	logical, if TRUE (default) the splines are plotted in color. If FALSE all splines are black lines.
digits	number of digits to the right of the decimal place to report for the value of each knot.
n	number of values to use to plot the splines, defaults to 100
...	not currently used

### See Also

[bsplines](#)

### Examples

```
bmat <- bsplines(seq(-3, 2, length = 1000), iknots = c(-2, 0, 0.2))
plot(bmat, show_xi = TRUE, show_x = TRUE)
plot(bmat, show_xi = FALSE, show_x = TRUE)
plot(bmat, show_xi = TRUE, show_x = FALSE) ## Default
plot(bmat, show_xi = FALSE, show_x = FALSE)
```

---

plot.cpr_cn	<i>Plotting Control Nets</i>
-------------	------------------------------

---

## Description

Three-dimensional plots of control nets and/or surfaces

## Usage

```
## S3 method for class 'cpr_cn'
plot(x, ..., xlab = "", ylab = "", zlab = "",
      show_net = TRUE, show_surface = FALSE, get_surface_args, net_args,
      surface_args, rgl = TRUE)
```

## Arguments

x	a cpr_cn object
...	common arguments which would be used for both the plot of the control net and the surface, e.g., xlim, ylim, zlim.
xlab, ylab, zlab	labels for the axes.
show_net	logical, show the control net
show_surface	logical, show the tensor product surface
get_surface_args	a list of arguments passed to the <a href="#">get_surface</a> call. This call generates the needed data sets used in the plotting.
net_args	arguments to be used explicitly for the control net. Ignored if show_net = FALSE.
surface_args	arguments to be used explicitly for the surface. Ignored if show_surface = FALSE.
rgl	If TRUE, the default, generate use <code>rgl::persp3d</code> to generate the graphics. If FALSE, use <code>plot3D::persp3D</code> to generate the graphics.

## Details

This plotting method generates three-dimensional plots of the control net, surface, or both, for a cpr\_cn objects. The three-dimensional plots are generated by either [persp3D](#) from the plot3D package or [persp3d](#) from the rgl package.

Building complex and customized graphics might be easier for you if you use [get\\_surface](#) to generate the needed data for plotting. See `vignette("cpr-pkg", package = "cpr")` for examples of building different plots.

For rgl graphics, the surface\_args and net\_args are lists of [rgl.material](#) and other arguments passed to [persp3d](#). Defaults are col = "black", front = "lines", back = "lines" for the net\_args and col = "grey20", front = "fill", back = "lines" for the surface\_args.

For plot3D graphics there are no defaults values for the net\_args and surface\_args.

**Value**

The result of the `get_surface` call is returned invisibly.

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

**See Also**

[plot.cpr\\_cp](#) for plotting control polygons and splines, [persp3d](#) and [rgl.material](#) for generating and controlling rgl graphics. [persp3D](#) for building plot3D graphics. [get\\_surface](#) for generating the data sets needed for the plotting methods.

**Examples**

```
## see vignette("cpr-pkg", package = "cpr")
```

---

plot.cpr\_cnr

*Control Net Reduction Plots*

---

**Description**

A collection of function for the inspection and evaluation of the control polygon reduction.

**Usage**

```
## S3 method for class 'cpr_cnr'  
plot(x, type = "rmse", from = 1, to, ...)
```

**Arguments**

x	a cpr_cnr object
type	type of diagnostic plot. "loglik" for the loglikelihood by degrees of freedom, "rmse" for root mean squared residuals by model index
from	the first index of x to plot
to	the last index of x to plot
...	ignored

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>



---

plot.cpr\_cpr                    *Control Polygon Reduction Plots*

---

**Description**

A collection of function for the inspection and evaluation of the control polygon reduction.

**Usage**

```
## S3 method for class 'cpr_cpr'  
plot(x, type = "cps", from = 1, to, ...)
```

**Arguments**

x	a cpr_cpr object
type	type of diagnostic plot. "cps" for control polygons, "loglik" for the loglikelihood by degrees of freedom, "rmse" for root mean squared residuals by degrees of freedom
from	the first index of x to plot
to	the last index of x to plot
...	args passed to cpr::plot.cpr_cp

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

**See Also**

[plot.cpr\\_cp](#)

---

plot.cpr\_influence\_of    *Plotting for cpr\_influence\_of objects*

---

**Description**

Plot method for cpr\_influence\_of objects

**Usage**

```
## S3 method for class 'cpr_influence_of'  
plot(x, ...)
```

**Arguments**

x a cpr\_influence\_of object  
... Arguments passed to [plot.cpr\\_cp](#)

**Value**

a ggplot2 graphic

**See Also**

[influence\\_of](#)

---

predict.cpr\_cp      *Model Prediction*

---

**Description**

Model prediction for cpr\_cp and cpr\_cn objects.

**Usage**

```
## S3 method for class 'cpr_cp'  
predict(object, newdata, ...)
```

**Arguments**

object a cpr\_cp or cpr\_cn object  
newdata a data.frame  
... passed to [predict](#) from the stats package.

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

---

print.cpr_bs	<i>Print bsplines</i>
--------------	-----------------------

---

**Description**

Print bsplines

**Usage**

```
## S3 method for class 'cpr_bs'
print(x, n = 6L, ...)
```

**Arguments**

x	a cpr_bs object.
n,	number of rows of the B-spline basis matrix to display, defaults to 6L.
...	not currently used.

---

refine_ordinate	<i>Knot Insertion, Removal, and Reinsertion</i>
-----------------	-------------------------------------------------

---

**Description**

Functions for the insertion, removal, and reinsertion of internal knots for B-splines.

**Usage**

```
refine_ordinate(x, xi, theta, order = 4L)

coarsen_ordinate(x, xi, theta, order = 4L)

hat_ordinate(x, xi, theta, order = 4L)

insertion_matrix(x, xi, order = 4L)
```

**Arguments**

x	the value of the knot to be inserted into the knot vector
xi	the (whole) knot vector, including the repeated boundary knots. Regardless of refinement or coarsening, this vector should be the 'reduced' vector such that x will be added to it. See details and examples.
theta	the ordinates of the control polygon vertices
order	the order of the B-spline, defaults to 4 for cubic splines

**Value**

numeric vectors

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

**Examples**

```
## Not run:
# See the vignette
vignette("cpr-pkg", package = "cpr")

## End(Not run)
```

---

spdg

*Simulated Prognanediol-glucuronid (PDG) Data*

---

**Description**

A Simulated data set based on the Study of Women's Health Across the Nation (SWAN) Daily Hormone Study (DHS).

**Usage**

spdg

**Format**

a `data.frame`, carries the `tbl_df` and `tbl` classes from `dplyr`. Variables in the data set:

**id** Subject ID

**age** Age, in years of the subject

**ttm** Time-to-menopause, in years

**ethnicity** Ethnicity, a factor with five levels: Caucasian, Black, Chinese, Hispanic, and Japanese

**bmi** Body Mass Index

**day\_from\_dlt** A integer value for the number of days from Day of Luteal Transition (DLT). The DLT is `day_from_dlt == 0`. Negative values indicate the follicular phase, positive values for the luteal phase.

**day\_of\_cycle** the day of cycle

**day** A scaled day-of-cycle between `[-1, 1]` with 0 for the DLT. See Details

**pdg** A simulated PDG value

**Details**

Progesterone-17β-glucuronide (PDG) is the urine metabolite of progesterone. This data set was simulated to have similar characteristics to a subset of the SWAN DHS data. The SWAN DHS data was the motivating data set for the method development that led to the cpr package. The DHS data cannot be made public, so this simulated data set has been provided for use in examples and instructions for use of the cpr package.

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

**Source**

This is simulated data. To see the script that generated the data set please visit <https://github.com/dewittpe/cpr> and look at the scripts in the data-raw directory.

**References**

Santoro, Nanette, et al. "Body size and ethnicity are associated with menstrual cycle alterations in women in the early menopausal transition: The Study of Women's Health across the Nation (SWAN) Daily Hormone Study." *The Journal of Clinical Endocrinology & Metabolism* 89.6 (2004): 2622-2631.

---

theta	<i>Extract Regression Coefficients for B-Splines and Tensor Products of B-splines</i>
-------	---------------------------------------------------------------------------------------

---

**Description**

An S3 method for extracting the regression coefficients of the bsplines and btensor terms. By default this uses `stats::coef` to extract all the regression coefficients. A specific method for `lmerMod` objects has been provided. If you are using a regression method which `stats::coef` will not return the regression coefficients, you'll need to define an S3 method for `stats::coef` to do so.

**Usage**

```
theta(fit)
```

**Arguments**

```
fit          a regression model fit
```

**Details**

This function is implicitly called in the `cpr::cp` and `cpr::cn` calls.

**Value**

the regression coefficients associated with terms with names containing either "bsplines" or "bten-sor".

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

**See Also**

[coef](#) [cp](#) [cn](#)

---

trimmed_quantile	<i>Trimmed Quantiles</i>
------------------	--------------------------

---

**Description**

For data  $X = x_1, x_2, \dots, x_n$ , with order statistics  $x(1), x(2), \dots, x(r)$  return the quantiles for a trimmed data set, e.g.,  $X \setminus x(1), x(r)$  (trim = 1), or  $X \setminus x(1), x(2), x(r-1), x(r)$  (trim = 2).

**Usage**

```
trimmed_quantile(x, trim = 1L, use_unique = TRUE, ...)
```

**Arguments**

<code>x</code>	a numeric vector
<code>trim</code>	defaults to 1, omitting the min and the max
<code>use_unique</code>	logical, if true (defaults), base the quantiles on unique values, if false, base the quantiles on all data, after trimming.
<code>...</code>	other arguments to pass to <code>stats::quantile</code>

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

**Examples**

```
trimmed_quantile(1:100, prob = 1:23 / 24, name = FALSE)

# Warning
# trimmed_quantile(1:100, trim = .3, prob = 1:23 / 24, name = FALSE)

# no warning
trimmed_quantile(1:100, trim = 3, prob = 1:23 / 24, name = FALSE)
```

---

update_bsplines	<i>Update bspline or btensor calls</i>
-----------------	----------------------------------------

---

**Description**

Update cpr\_bs and cpr\_bt objects alone or within cpr\_cp and cpr\_cn objects.

**Usage**

```
update_bsplines(object, ..., evaluate = TRUE)
```

```
update_btensor(object, ..., evaluate = TRUE)
```

**Arguments**

object	an object to update.
...	things to update, expected to be iknots, df, bknots, or order.
evaluate	If true evaluate the new call else return the call.

**Author(s)**

Peter DeWitt <dewittpe@gmail.com>

**See Also**

[update](#), [bsplines](#), [btensor](#)

**Examples**

```
##### Updating a cpr_bs object #####
# construct a B-spline basis
bmat <- bsplines(seq(1, 10, length = 15), df = 5, order = 3)

# look at the structure of the basis
str(bmat)

# change the order
str(update_bsplines(bmat, order = 4))

# change the order and the degrees of freedom
str(update_bsplines(bmat, df = 12, order = 4))

##### Updating a cpr_bt object #####
# construct a tensor product
tpmat <- btensor(list(x1 = seq(0, 1, length = 10), x2 = seq(0, 1, length = 10)),
                 df = list(4, 5))

tpmat
```

```

# update the degrees of freedom
update_btensor(tpmat, df = list(6, 7))

##### Updating bsplines or btensor on the right and side of a formula #####

f1 <- y ~ bsplines(x, df = 14) + var1 + var2
f2 <- y ~ btensor(x = list(x1, x2), df = list(50, 31), order = list(3, 5)) + var1 + var2

update_bsplines(f1, df = 13, order = 5)
update_btensor(f2, df = list(13, 24), order = list(3, 8))

##### Updating a cpr_cp object #####
data(spdg, package = "cpr")
init_cp <- cp(pdg ~ bsplines(day, df = 30) + age + ttm, data = spdg)
updt_cp <- update_bsplines(init_cp, df = 5)

##### Updating a cpr_cn object #####
init_cn <- cn(pdg ~ btensor(list(day, age), df = list(30, 4)) + ttm, data = spdg)
updt_cn <- update_btensor(init_cn, df = list(30, 2), order = list(3, 2))

```

---

wiggle

*Wiggleness of a Spline function*

---

## Description

Calculate the integral of the squared second derivative of the spline function.

## Usage

```
wiggle(object, lower, upper, stop.on.error = FALSE, ...)
```

## Arguments

object	a cpr_cp object
lower	the lower limit of the integral
upper	the upper limit of the integral
stop.on.error	default to FALSE, see <a href="#">integrate</a> .
...	arguments passed to <code>stats::integrate</code>

## Details

$$\int \left( \frac{d^2}{dx^2} f(x) \right)^2 dx.$$

## Author(s)

Peter DeWitt <dewittpe@gmail.com>



*wiggle*

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**See Also**

[cp integrate](#)

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