

Package ‘splmm’

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Type Package

Title Simultaneous Penalized Linear Mixed Effects Models

Version 1.1.3

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Description Contains functions that fit linear mixed-effects models for high-dimensional data ($p \gg n$) with penalty for both the fixed effects and random effects for variable selection.
The details of the algorithm can be found in Luoying Yang PhD thesis (Yang and Wu 2020). The algorithm implementation is based on the R package 'Immlasso'.
Reference: Yang L, Wu TT (2020). Model-Based Clustering of Longitudinal Data in High-Dimensionality. Unpublished thesis.

License GPL-3

Imports Rcpp (>= 1.0.1), emulator, miscTools, penalized, ggplot2, gridExtra, plot3D, MASS

LinkingTo Rcpp, RcppArmadillo

NeedsCompilation yes

Depends R (>= 3.5.0)

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splmm-package	<i>Simultaneous Penalized Linear Mixed Effects Models</i>
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Description

Contains functions that fit linear mixed-effects models for high-dimensional data ($p \gg n$) with penalty for both the fixed effects and random effects for variable selection. The details of the algorithm can be found in Luoying Yang PhD thesis (Yang and Wu 2020). The algorithm implementation is based on the R package 'lmmlasso'. Reference: Yang L, Wu TT (2020). Model-Based Clustering of Longitudinal Data in High-Dimensionality. Unpublished thesis.

Details

The DESCRIPTION file:

```

Package:      splmm
Type:        Package
Title:       Simultaneous Penalized Linear Mixed Effects Models
Version:     1.1.3
Date:       2021-08-17
Authors@R:  c(person(given = "Luoying", family = "Yang", role = c("aut", "cre"), email = "lyang19@u.rochester.edu
Maintainer:  Luoying Yang <lyang19@u.rochester.edu>
Description: Contains functions that fit linear mixed-effects models for high-dimensional data (p>>n) with penalty for
License:     GPL-3
Imports:    Rcpp (>= 1.0.1), emulator, miscTools, penalized, ggplot2, gridExtra, plot3D, MASS
LinkingTo:  Rcpp, RcppArmadillo
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Depends:    R (>= 3.5.0)
Author:     Luoying Yang [aut, cre], Tong Tong Wu [aut]

```

Index of help topics:

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                Dataset
plot.splmm     Plot the tuning results of a 'splmm.tuning'
                object
plot3D.splmm   3D Plot the tuning results of a "splmm.tuning"
                object when tuning over both lambda 1 and

```

	lambda 2 grids
print.splmm	Print a short summary of a splmm object.
simulated_data	Dataset simulated for toy example
splmm	Function to fit linear mixed-effects model with double penalty for fixed effects and random effects
splmm-package	Simultaneous Penalized Linear Mixed Effects Models
splmmControl	Options for the 'splmm' Algorithm
splmmTuning	Tuning function of "splmm" object
summary.splmm	Summarize an 'splmm' object

Contains functions that fit linear mixed-effects models for high-dimensional data ($p \gg n$) with penalty for both the fixed effects and random effects for variable selection.

Author(s)

NA

Maintainer: Luoying Yang <lyang19@u.rochester.edu>

References

Luoying Yang PhD thesis

SCHELLDORFER, J., BUHLMANN, P. and DE GEER, S.V. (2011), Estimation for High-Dimensional Linear Mixed-Effects Models Using L1-Penalization. *Scandinavian Journal of Statistics*, 38: 197-214. doi:10.1111/j.1467-9469.2011.00740.x

Examples

```
## Use splmm on the Kenya school cognitive data set

data(cognitive)

x <- model.matrix(ravens ~schoolid+treatment+year+sex+age_at_time0
                 +height+weight+head_circ+ses+mom_read+mom_write
                 +mom_edu, cognitive)

z <- x

fit <- splmm(x=x,y=cognitive$ravens,z=z,grp=cognitive$id,lam1=0.1,
            lam2=0.1,penalty.b="lasso", penalty.L="lasso")
summary(fit)
```

Description

In the Kenya school lunch intervention, children were given one of four school lunch interventions: meat, milk, calorie, or control. The first three groups were fed a school lunch of a stew called githeri supplemented with either meat, milk, or oil to create a lunch with a given caloric level, while the control group did not receive a lunch. Three schools were randomized to each group and the lunch program is the same for all children within a school. The data is available in *modeling-longitudinal-data-rob-weiss* and is broken up into sub data sets from four domains: Anthropometry, Cognitive, Morbidity, and Nutrition. We will be using the cognitive dataset for analyzing how the cognition level of the school children change over time and how the change is associated with other variables. The main cognitive measure is Raven's colored progressive matrices (Raven's), a measure of cognitive ability. There are three additional response variables: arithmetic score (arithmetic), verbal meaning (vmeaning), and total digit span score (dstotal) where digit span is a test of memory while others are considered measures of intelligence or education. The cognitive measurement baseline was taken prior to the lunch program onset and measurements were assessed at up to five times, called rounds, for each subject. More information about this dataset please see the reference:

Robert E Weiss. Modeling longitudinal data. Springer Science & Business Media, 2005.

Usage

```
data(cognitive)
```

Format

A data frame of 1562 observations and 26 variables.

Grouping variable. Unique ID for each subject.

schboldid School id 1-12.

treatment Calorie, meat, milk, control

rn round.

year Time in years from baseline.

revans Raven's colored matrices score.

arithmetic Arithmetic score.

vmeaning Verbal meaning.

dstotal Total digit span score.

sex Girl or Boy.

age_at_time0 age at baseline.

height height at baseline.

weight weight at baseline.

head_circ Head circumference at baseline.

ses Socio-Economic Status score.

mom_read Mother's reading test.

mom_write Mother's writing test.

mom_edu Mother's years of educations.

morbscore Morbidity score: none/mild/severe.

complete Logical variable specifying whether the subject has all five rounds. 1-Yes, 0-No.

rneone Logical variable specifying whether the observation is the baseline. 1-round one (baseline), 0-not round one.

relmonth Time in months from baseline.

Examples

```
data(cognitive)
```

plot.splmm	<i>Plot the tuning results of a splmm.tuning object</i>
------------	---

Description

This function inputs an `splmm.tuning` object and plot the model selection criterion values over the tuning parameters grid.

Usage

```
## S3 method for class 'splmm'
plot(x, ...)
```

Arguments

<code>x</code>	a 'splmm.tuning' object
<code>...</code>	not used

Value

A line plot of BIC, AIC, BICC, EBIC values against lam1 or lam2 depending on the inout.

See Also

`plot.splmm`

Examples

```

data(cognitive)

x <- model.matrix(ravens ~schoolid+treatment+year+sex+age_at_time0
                 +height+weight+head_circ+ses+mom_read+mom_write
                 +mom_edu, cognitive)
z <- x

## Tuning over lambda1 grid
lam1 = seq(0.1,0.5,0.1)
lam2 = 0.1
fit1 <-splmmTuning(x=x,y=cognitive$ravens,z=z,grp=cognitive$id,lam1=lam1,
lam2=lam2,penalty.b="scad", penalty.L="scad")
plot.splmm(fit1)

```

plot3D.splmm	<i>3D Plot the tuning results of a 'splmm.tuning' object when tuning over both lambda 1 and lambda 2 grids</i>
--------------	--

Description

This function inputs an 'splmm.tuning' object and plot the model selection criterion values in a 3D plot over the lambda 1 and lambda 2 tuning parameters grid.

Usage

```

## S3 method for class 'splmm'
plot3D(x, criteria=c("BIC","AIC","BICC","EBIC"),type=c("line","surface"),...)

```

Arguments

x	a 'splmm.tuning' object with both lam1.tuning=TRUE and lam2.tuning=TRUE
criteria	A parameter specifying whether the criteria value the user want to plot is BIC, AIC, BICC or EBIC. The default is BIC
type	A parameter specifying which type of 3D plot to use for plotting. Currently the available options include line plot and surface plot. The default is surface plot.
...	not used

Value

A 3D line/surface plot of BIC/AIC/BICC/EBIC values against lam1 and lam2.

See Also

plot3D

Examples

```
data(cognitive)

x <- model.matrix(ravens ~schoolid+treatment+year+sex+age_at_time0
                  +height+weight+head_circ+ses+mom_read+mom_write
                  +mom_edu, cognitive)

z <- x

## Tuning over lambda1 grid and lambda2 grid
lam1 = seq(0.1,0.5,0.1)
lam2 = seq(0.1,0.5,0.1)
fit1 <-splmmTuning(x=x,y=cognitive$ravens,z=z,grp=cognitive$id,lam1=lam1,
lam2=lam2,penalty.b="scad", penalty.L="scad")
plot3D.splmm(fit1)
```

`print.splmm`*Print a short summary of a splmm object.*

Description

Prints a short summary of an 'splmm' object comprising information about the nonzero fixed-effects coefficients and the nonzero random effect variance components.

Usage

```
## S3 method for class 'splmm'
print(x, ...)
```

Arguments

x	a 'splmm' object
...	not used

Value

No return value, a print-out of a 'splmm' object's short summary is produced.

See Also

print.splmm

Examples

```
data(simulated_data)

set.seed(144)
fit = splmm(x=simulated_data$x,y=simulated_data$y,
z=simulated_data$z,grp=simulated_data$grp,
lam1=0.1,lam2=0.01, penalty.b="scad", penalty.L="scad")
print(fit)

data(cognitive)

x <- model.matrix(ravens ~schoolid+treatment+year+sex+age_at_time0
+height+weight+head_circ+ses+mom_read+mom_write
+mom_edu, cognitive)
z <- x

fit <- splmm(x=x,y=cognitive$ravens,z=z,grp=cognitive$id,lam1=0.1,
lam2=0.1,penalty.b="scad", penalty.L="scad")
print(fit)
```

simulated_data

Dataset simulated for toy example

Description

A toy dataset simulated for demonstration for the splmm function.

Usage

```
data(simulated_data)
```

Format

y Response variable.
x Fixed-effects design matrix.
z Random-effects design matrix
grp Subject ID.

Examples

```
data(simulated_data)
```

splmm	<i>Function to fit linear mixed-effects model with double penalty for fixed effects and random effects</i>
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Description

All the details of the algorithm can be found in the manuscript.

Usage

```
splmm(x, y, z, grp, lam1, lam2, nonpen.b=1, nonpen.L=1, penalty.b=c("lasso", "scad"),
      penalty.L=c("lasso", "scad"), CovOpt=c("nlminb", "optimize"),
      standardize=TRUE, control=splmmControl())
```

```
## Default S3 method:
```

```
splmm(x, y, z, grp, lam1, lam2, nonpen.b=1, nonpen.L=1, penalty.b=c("lasso", "scad"),
      penalty.L=c("lasso", "scad"), CovOpt=c("nlminb", "optimize"),
      standardize=TRUE, control=splmmControl())
```

Arguments

x	matrix of dimension $N \times p$ including the fixed-effects covariables. An intercept has to be included in the first column as $(1, \dots, 1)$.
y	response variable of length N .
z	random effects matrix of dimension $N \times q$. It has to be a matrix, even if $q=1$.
grp	grouping variable of length N
lam1	regularization parameter for fixed effects penalization.
lam2	regularization parameter for random effects penalization.
nonpen.b	Index of indices of fixed effects not penalized. The default value is 1, which means the fixed intercept is not penalized
nonpen.L	Index of indices of random effects not penalized. The default value is 1, which means the random intercept is not penalized
penalty.b	The penalty method for fixed effects penalization. Currently available options include LASSO penalty and SCAD penalty.
penalty.L	The penalty method for random effects penalization. Currently available options include LASSO penalty and SCAD penalty.
CovOpt	which optimization routine should be used for updating the variance parameter. The available options include optimize and nlminb. nlminb uses the estimate of the last iteration as a starting value. nlminb is faster if there are many Gauss-Seidel iterations.
standardize	A logical parameter specifying whether the fixed effects matrix x and random effects matrix z should be standardized such that each column has mean 0 and standard deviation 1. The default value is TRUE
control	control parameters for the algorithm and the Armijo Rule, see 'splmmControl' for the details

Value

A 'splmm' object is returned, for which coef, resid, fitted, print, summary methods exist.

data	data set used for fitting the model, as a list with four components: x, y, z, grp (see above)
coefInit	list of the starting values for beta, random effects covariance structure, and variance structure
penalty.b	The penalty method for fixed effects penalization.
penalty.L	The penalty method for random effects penalization.
nonpen.b	Index of indices of fixed effects not penalized.
nonpen.L	Index of indices of random effects not penalized.
lambda1	regularization parameter for fixed effects penalization scaled by the number of subjects.
lambda2	regularization parameter for random effects penalization the number of subjects.
sigma	standard deviation $\hat{\sigma}$ of the errors
D	The estimates of the random effects covariance matrix \hat{D} .
Lvec	Vectorized \hat{L} , the lower triangular matrix of \hat{D} from Cholesky Decomposition.
coefficients	estimated fixed-effects coefficients $\hat{\beta}$
random	vector with random effects, sorted by groups
ranef	vector with random effects, sorted by effect
u	vector with the standardized random effects, sorted by effect
fixef	estimated fixed-effects coefficients $\hat{\beta}$
fitted.values	The fitted values $\hat{y} = \hat{X}\beta + Z\hat{b}_i$
residuals	raw residuals $y - \hat{y}$
corD	Correlation matrix of the random effects
logLik	value of the log-likelihood function
deviance	deviance = $-2 * \logLik$
npar	Number of parameters. Corresponds to the cardinality of the set of nonzero coefficients plus the number of nonzero variance in D
aic	AIC
bic	BIC
bicc	Modified BIC defined by Wang et al (2009)
ebic	Extended BIC defined by Chen and Chen (2008)
converged	Does the algorithm converge? 0: correct convergence ; an odd number means that maxIter was reached ; an even number means that the Armijo step was not succesful. For each unsuccessfull Armijo step, 2 is added to converged. If converged is large compared to the number of iterations counter, you may increase maxArmijo.
counter	The number of iterations used.

stopped	logical indicating whether the algorithm stopped due to too many parameters, if yes need to increase lam1 or lam2
CovOpt	optimization routine
control	see splmmControl
objective	Value of the objective function at the final estimates
call	call

Examples

```
### Use splmm for a toy dataset.

data(simulated_data)

set.seed(144)
fit = splmm(x=simulated_data$x,y=simulated_data$y,
z=simulated_data$z,grp=simulated_data$grp,
lam1=0.1,lam2=0.01, penalty.b="scad", penalty.L="scad")
summary(fit)

## Use splmm on the Kenya school cognitive data set

data(cognitive)

x <- model.matrix(ravens ~schoolid+treatment+year+sex+age_at_time0
+height+weight+head_circ+ses+mom_read+mom_write
+mom_edu, cognitive)

z <- x

fit <- splmm(x=x,y=cognitive$ravens,z=z,grp=cognitive$id,lam1=0.1,
lam2=0.1,penalty.b="lasso", penalty.L="lasso")
summary(fit)
```

splmmControl

Options for the 'splmm' Algorithm

Description

Definition of various kinds of options in the algorithm.

Usage

```
splmmControl(tol=10-4,trace=1,maxIter=1000,maxArmijo=20,number=5,a_init=1,
delta=0.1,rho=0.001,gamma=0,lower=10-6,upper=108,seed=532,VarInt=c(0,10),
CovInt=c(-5,5),thres=10-4)
```

Arguments

tol	convergence tolerance
trace	integer. 1 prints no output, 2 prints warnings, 3 prints the current function values and warnings (not recommended)
maxIter	maximum number of (outer) iterations
maxArmijo	maximum number of steps to be chosen in the Armijo Rule. If the maximum is reached, the algorithm continues with optimizing the next coordinate.
number	integer. Determines the active set algorithm. The zero fixed-effects coefficients are only updated each number iteration. It may be that a smaller number increases the speed of the algorithm. Use $0 \leq number \leq 5$.
a_init	α_{init} in the Armijo step. See Schelldorfer et. al. (2010).
delta	δ in the Armijo step. See Schelldorfer et. al. (2010)
rho	ρ in the Armijo step. See Schelldorfer et. al. (2010)
gamma	γ in the Armijo step. See Schelldorfer et. al. (2010)
lower	lower bound for the Hessian
upper	upper bound for the Hessian
seed	set.seed for calculating the starting value, which performs a 10-fold cross-validation.
VarInt	Only for opt="optimize". The interval for the variance parameters used in "optimize". See help("optimize")
CovInt	Only for opt="optimize". The interval for the covariance parameters used in "optimize". See help("optimize")
thres	If a variance or covariance parameter has smaller absolute value than thres, the parameter is set to exactly zero.

Details

For the Armijo step parameters, see Bertsekas (2003)

Value

Exactly the same as arguments.

splmmTuning

Tuning function of 'splmm' object

Description

This function fits 'splmm' function over grids of lambda1 and/or lambda2 and determine the best fit model based on model selection information criterion. The function takes a scalar or a grid of lambda1 and/or lambda2 and determine the optimal tuning parameter value for the best model fit. If both lambda1 and lambda2 are inputted as scalars, an 'splmm' object is returned; if either or both lambda1 and lambda2 are inputted as grids, an 'splmm.tuning' object is returned. Currently the model selection criterion include AIC and BIC, and BIC is used to determine the optimal model.

Usage

```
splmmTuning(x, y, z, grp, lam1.seq, lam2.seq, nonpen.b=1, nonpen.L=1,
            penalty.b=c("lasso", "scad"),
            penalty.L=c("lasso", "scad"),
            CovOpt=c("nlminb", "optimize"),
            standardize=TRUE, control=splmmControl())
```

Arguments

x	matrix of dimension $N \times p$ including the fixed-effects covariables. An intercept has to be included in the first column as (1,...,1).
y	response variable of length N .
z	random effects matrix of dimension $N \times q$. It has to be a matrix, even if $q=1$.
grp	grouping variable of length N
lam1.seq	a grid of regularization parameter for fixed effects penalization, could be a scalar if no need to tune.
lam2.seq	a grid of regularization parameter for random effects penalization, could be a scalar if no need to tune.
nonpen.b	Index of indices of fixed effects not penalized. The default value is 1, which means the fixed intercept is not penalized.
nonpen.L	Index of indices of random effects not penalized. The default value is 1, which means the random intercept is not penalized.
penalty.b	The penalty method for fixed effects penalization. Currently available options include LASSO penalty and SCAD penalty.
penalty.L	The penalty method for random effects penalization. Currently available options include LASSO penalty and SCAD penalty.
CovOpt	which optimization routine should be used for updating the variance parameter. The available options include optimize and nlminb. nlminb uses the estimate of the last iteration as a starting value. nlminb is faster if there are many Gauss-Seidel iterations.
standardize	A logical parameter specifying whether the fixed effects matrix x and random effects matrix z should be standardized such that each column has mean 0 and standard deviation 1. The default value is TRUE.
control	control parameters for the algorithm and the Armijo Rule, see 'splmmControl' for the details.

Value

A 'splmm.tuning' object is returned, for which plot method exist.

lam1.seq	lambda1 grid used for tuning. Only available when lambda1 is inputted as a vector.
lam2.seq	lambda2 grid used for tuning. Only available when lambda2 is inputted as a vector.

BIC.lam1	A vector of BIC values of splmm models fitting over a lambda1 grid.
BIC.lam2	A vector of BIC values of splmm models fitting over a lambda2 grid.
fit.BIC	An array of BIC values of splmm models fitting over lambda 1 grid x lambda2 grid.
AIC.lam1	A vector of AIC values of splmm models fitting over a lambda1 grid.
AIC.lam2	A vector of AIC values of splmm models fitting over a lambda2 grid.
fit.AIC	An array of AIC values of splmm models fitting over lambda 1 grid x lambda2 grid.
BICC.lam1	A vector of BICC values of splmm models fitting over a lambda1 grid.
BICC.lam2	A vector of BICC values of splmm models fitting over a lambda2 grid.
fit.BICC	An array of BICC values of splmm models fitting over lambda 1 grid x lambda2 grid.
EBIC.lam1	A vector of EBIC values of splmm models fitting over a lambda1 grid.
EBIC.lam2	A vector of EBIC values of splmm models fitting over a lambda2 grid.
fit.EBIC	An array of EBIC values of splmm models fitting over lambda 1 grid x lambda2 grid.
min.BIC	The minimum BIC value from tuning over a grid. This is only available when either lambda1 or lambda2 is a scalar.
min.AIC	The minimum AIC value from tuning over a grid. This is only available when either lambda1 or lambda2 is a scalar.
min.BICC	The minimum BICC value from tuning over a grid. This is only available when either lambda1 or lambda2 is a scalar.
min.EBIC	The minimum EBIC value from tuning over a grid. This is only available when either lambda1 or lambda2 is a scalar.
best.model	The index of the optimal model. This is only available when either lambda1 or lambda2 is a scalar.
best.fit	The optimal model chosen by the minimum BIC as an splmm object.
min.lam1	lambda1 value that results in the optimal model. This is only available when input lambda1 is a vector.
min.lam2	lambda2 value that results in the optimal model. This is only available when input lambda2 is a vector.
lam1.tuning	A logical parameter specifying if tuning is performed over lamdbda1 grid. lam1.tuning=TRUE if input lambda1 is a vector.
lam2.tuning	A logical parameter specifying if tuning is performed over lamdbda2 grid. lam1.tuning=TRUE if input lambda2 is a vector.

Examples

```
data(cognitive)

x <- model.matrix(ravens ~schoolid+treatment+year+sex+age_at_time0
```

```

+height+weight+head_circ+ses+mom_read+mom_write
+mom_edu, cognitive)

z <- x

## Tuning over lambda1 grid
lam1 = seq(0.1,0.5,0.1)
lam2 = 0.1
fit1 <-splmmTuning(x=x,y=cognitive$ravens,z=z,grp=cognitive$id,lam1.seq=lam1,
lam2.seq=lam2,penalty.b="scad", penalty.L="scad")
plot.splmm(fit1)

## Tuning over lambda2 grid
lam1 = 0.1
lam2 = seq(0.1,0.5,0.1)
fit2 <-splmmTuning(x=x,y=cognitive$ravens,z=z,grp=cognitive$id,lam1.seq=lam1,
lam2.seq=lam2,penalty.b="scad", penalty.L="scad")
plot.splmm(fit2)

## Tuning over both lambda1 and lambda2 grid
lam1 = seq(0.1,0.5,0.2)
lam2 = seq(0.1,0.5,0.2)
fit3 <-splmmTuning(x=x,y=cognitive$ravens,z=z,grp=cognitive$id,lam1.seq=lam1,
lam2.seq=lam2,penalty.b="scad", penalty.L="scad")
plot.splmm(fit3)

```

summary.splmm

Summarize an 'splmm' object

Description

Providing an elaborate summary of a 'splmm' object.

Usage

```
## S3 method for class 'splmm'
summary(object, ...)
```

Arguments

```
object      a 'splmm' object
...         not used.
```

Details

This functions shows a detailed summary of a 'splmm' object.

Value

No return value, a print-out of a 'splmm' object's detailed summary is produced.

Examples

```
data(simulated_data)
```

```
set.seed(144)
fit = splmm(x=simulated_data$x,y=simulated_data$y,
z=simulated_data$z,grp=simulated_data$grp,
lam1=0.1,lam2=0.01, penalty.b="scad", penalty.L="scad")
summary(fit)
```

```
data(cognitive)
```

```
x <- model.matrix(ravens ~schoolid+treatment+year+sex+age_at_time0
+height+weight+head_circ+ses+mom_read+mom_write
+mom_edu, cognitive)
z <- x

fit <- splmm(x=x,y=cognitive$ravens,z=z,grp=cognitive$id,lam1=0.1,lam2=0.1,
penalty.b="scad", penalty.L="scad")
summary(fit)
```


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